The Size-Centrality Relationship in Production Networks*

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Abstract

Standard production network models with only technology shocks predict that industry size and centrality move in opposite directions. Yet in UK data, they co-move positively: larger industries are more central as input suppliers, and industries that grow tend to become more central. We trace this tension to the elasticity of substitution across intermediate inputs: when it is below unity, as we estimate, technology shocks generate a negative size-centrality relationship. Demand-side shocks resolve the puzzle. Applying the framework to the UK post-2010 productivity slowdown, we find that manufacturing-specific shocks more than account for the slowdown, while common shocks partially offset it.

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1 Introduction

How shocks propagate through production networks and generate aggregate impacts depends on two characteristics of the industries that are hit: their *size* and their *centrality* as input suppliers (e.g., Hulten, 1978; Acemoglu et al., 2012; Baqaee and Farhi, 2019; Baqaee and Farhi, 2020). But do large industries also tend to be central? And when an industry grows, does it become more central in the network? Using UK data, we show that the answer is yes on both counts—and that this fact is difficult to reconcile with standard models of production networks.

This positive size-centrality relationship, which holds both in the long run and over the business cycle, has sharp implications for the types of shocks present in the economy. Specifically, we demonstrate both analytically and quantitatively that canonical production network models featuring only technology shocks cannot generate this relationship when intermediate inputs are gross complements, as the data suggest. Introducing demand (preference) shocks resolves this tension. We apply our framework to the UK's post-2010 productivity growth slowdown and find that idiosyncratic shocks to the manufacturing sector have been the dominant driver.

Our analysis proceeds in four steps. First, using input-output data for 79 UK industries over 1997–2019, we document the empirical size-centrality relationship. We measure industry size by real gross output or Domar weights, and centrality by first-order (weighted) outdegrees. The positive association holds both in time-averaged levels and in year-on-year growth rates. The latter finding that an industry's size and centrality tend to move together over time is particularly informative for our analysis, as it confirms that the input-output network responds endogenously to shocks, a feature central to our theoretical results on the identification of shock types.

Second, we set up a multisector general equilibrium model with a production network, building on Atalay (2017). The model features both technology (supply) and preference (demand) shocks, and industries aggregate their intermediate inputs using a CES technology with elasticity of substitution ε_M. We simplify the model to derive analytical results on how industry size and centrality respond to each type of shock. The key insight is that the direction of the size-centrality co-movement depends on the interaction between ε_M and the nature of the shock. When $\varepsilon_M < 1$, which is the empirically relevant case, a positive technology shock to an industry raises its real output but *lowers* its centrality, because the resulting price decline reduces the industry's share in other industries' input expenditure (the "price effect" dominates the "quantity effect"). This generates a negative size-centrality relationship. By contrast, a positive preference shock raises both the industry's output and its centrality, generating a positive relationship. Therefore, models with only technology shocks predict a counterfactually negative relationship between real output and centrality outside of the steady state.

Third, we calibrate the full dynamic model to UK data, estimate $\varepsilon_M \approx 0.35$ using an instrumental variables strategy based on military spending, and filter out the complete set of technology and preference shocks. The model-implied size-centrality relationship, computed from the filtered shocks, confirms the analytical predictions: technology shocks alone produce a strongly negative correlation between real output and centrality, while the combination of both shock types generates the positive correlation observed in the data.

Fourth, we apply the framework to analyse the UK's productivity growth slowdown following the 2008–09 recession. Our production network model allows us to go beyond standard accounting decompositions (e.g., Tenreyro, 2018) by identifying the underlying shocks—both idiosyncratic and common—and tracing their propagation through the network. We find that idiosyncratic shocks to the manufacturing sector account for the bulk of the aggregate slowdown. Common technology shocks, by contrast, have made a modestly positive contribution to the post-crisis change in aggregate productivity growth, partially offsetting the drag from manufacturing. Manufacturing's outsized role reflects both its large Domar weight and its high centrality as an input supplier: shocks to manufacturing propagate powerfully downstream through the network, consistent with the mechanisms emphasised in Sections 3 and 4.

Related Literature. This paper contributes to two strands of the literature. First, we contribute to the literature on the role of production networks in economic fluctuations. On the theoretical side, a central question is how industry size and centrality shape the macroeconomic impact of microeconomic shocks.[1] Hulten (1978) shows that in an efficient economy, the industry's sales as a fraction of GDP—a measure of its size—is a sufficient statistic for how a productivity shock to that industry affects aggregate TFP, up to a first order. Baqaee and Farhi (2019) extend Hulten's theorem to the second order, showing that an industry's centrality in the production network plays an important role in shaping the second-order impact of sectoral productivity shocks on aggregate TFP. Baqaee and Farhi (2020) further show that in an inefficient economy, both size and centrality matter even at the first order.[2] We contribute to this literature by studying the relationship between these two key objects both empirically and theoretically. First, using UK input-output data, we document a new empirical fact: industry size and centrality co-move positively, both in

[1]Some examples include Acemoglu et al. (2012), Carvalho and Gabaix (2013), Jones (2013), Acemoglu, Ozdaglar, and Tahbaz-Salehi (2017), Grassi (2017), Baqaee (2018), Liu (2019), Bigio and La'O (2020), Boehm and Oberfield (2020), and Altinoglu (2021). See Carvalho and Tahbaz-Salehi (2019) and Baqaee and Rubbo (2023) for surveys of this literature.

[2]While the literature typically focuses on supply-side shocks, Acemoglu, Akcigit, and Kerr (2016) analyse demand-side shocks and show that their propagation through production networks differs from that of technology shocks.

the long run and over the business cycle. Second, we provide an analytical characterisation of how industry size and centrality respond to both technology and preference shocks, showing that the sign of the size-centrality relationship depends on the interaction between the elasticity of substitution across intermediates and the nature of the shock.

On the quantitative side, a long tradition of multisector models has studied whether and how sectoral shocks translate into aggregate fluctuations (Long and Plosser, 1983; Horvath, 1998, 2000; Dupor, 1999; Carvalho, 2010; Di Giovanni, Levchenko, and Méjean, 2014). Our model builds most directly on Foerster, Sarte, and Watson (2011) and Atalay (2017), who develop dynamic multisector models with input-output linkages and use them to filter out sectoral shocks from data on industries' output. Both papers focus exclusively on technology shocks and rely on industries' output growth to back out the shocks. We extend this approach in two directions. First, we introduce preference shocks alongside technology shocks, motivated by our finding that technology shocks alone generate a counterfactually negative size-centrality relationship when industries cannot easily substitute across their input suppliers. Second, we use data on both value-added and prices—rather than output alone as in previous work—to separately identify the two types of shocks. Our finding that demand shocks are essential for matching the empirical size-centrality relationship is, to our knowledge, new.[3]

Second, our application to the UK productivity slowdown contributes to the literature on the post-crisis "productivity puzzle." Since the 2008–09 recession, UK labour productivity has fallen persistently below its pre-crisis trend, prompting a large body of research into its causes.[4] At the sectoral level, Riley, Rosazza-Bondibene, and Young (2015) and Tenreyro (2018) document that manufacturing and finance account for the bulk of the slowdown in an accounting sense. These decompositions identify *where* the slowdown occurred but not *why*: they do not distinguish whether the observed slowdowns in manufacturing and finance reflect sector-specific shocks, common shocks, or shocks originating in other industries and transmitted through the production network. We address this gap by using our production network model to identify the underlying shocks—both

[3] On the empirical side, a growing body of work has traced the propagation of shocks through production networks using natural experiments and microdata. Acemoglu, Akcigit, and Kerr (2016) provide broad evidence on upstream and downstream propagation using US input-output data. Barrot and Sauvagnat (2016) and Boehm, Flaaen, and Pandalai-Nayar (2019) document the downstream propagation of firm-level shocks using natural disasters, and Carvalho et al. (2021) quantify the aggregate effects of supply chain disruptions following the 2011 Great East Japan Earthquake. Bernard, Dhyne, Magerman, Manova, and Moxnes (2022) study the origins of firm size heterogeneity in Belgian firm-to-firm transaction data, finding that downstream network features—especially the number of customers—account for the vast majority of firm size dispersion.

[4] See Barnett et al. (2014) for an early overview. Proposed explanations include capital shallowing (Pessoa and Van Reenen, 2014), mismeasurement of intangible investment (Goodridge, Haskel, and Wallis, 2013), and impaired resource reallocation in the aftermath of the financial crisis (Riley, Rosazza-Bondibene, and Young, 2015). Goodridge, Haskel, and Wallis (2018) argue that the slowdown is fundamentally a TFP puzzle, since it cannot be accounted for by changes in labour or capital inputs.

idiosyncratic and common—and trace their propagation across industries. Our finding that manufacturing-specific shocks are the primary driver provides a structural interpretation of these earlier decompositions: manufacturing's persistent drag on aggregate productivity reflects not only its own decline but also the downstream propagation of this decline to its many customer industries.

Outline. The rest of the paper is organised as follows. Section 2 documents the empirical size-centrality relationship using UK data. Section 3 develops the model and derives analytical results on the size-centrality relationship. Section 4 takes the model to the data and confirms the analytical findings quantitatively. Section 5 applies the framework to the UK productivity slowdown. Section 6 concludes.

2 Data and Stylised Facts

In this section, we document the empirical relationship between industry size and centrality. We establish that larger industries tend to be more central in the UK production network, both in the long run (corresponding to the steady state in our model) and over the business cycle (corresponding to deviations from it). This positive size-centrality relationship serves as the key motivating fact for the model we develop in Section 3.

2.1 Data and Definitions

We study the UK production network using the input-output supply and use tables provided by the Office for National Statistics, which record intermediate input flows between industries at the 2-digit SIC07 level over 1997–2019.[5] For a given year, these tables record the nominal value of intermediate input flows between all pairs of industries. We construct the (weighted) input-output matrix $\mathbf{W}_t$, whose typical element ω_{ijt} gives the share of industry i's total intermediate input expenditure accounted for by inputs sourced from industry j:

$$\omega_{ijt} := \frac{P_{jt} M_{ijt}}{\sum_{k=1}^{N} P_{kt} M_{ikt}}, \tag{1}$$

where P_{jt} denotes the price of good j in period t and M_{ijt} denotes the quantity of good j used in the production of good i.

We measure industry *size* in two ways: (i) real gross output (Q_{jt}), which captures absolute size, and (ii) the Domar weight (λ_{jt}), defined as the ratio of industry j's nominal gross output to nominal GDP, which captures relative size. We measure industry *centrality*

[5] No comprehensive firm-level dataset on input-output linkages is available for the UK. See Appendix A.1 for a detailed description of the data.

using the first-order (weighted) outdegree of industry j:

$$D_{jt}^{\text{out}} := \sum_{i=1}^{N} \omega_{ijt}, \tag{2}$$

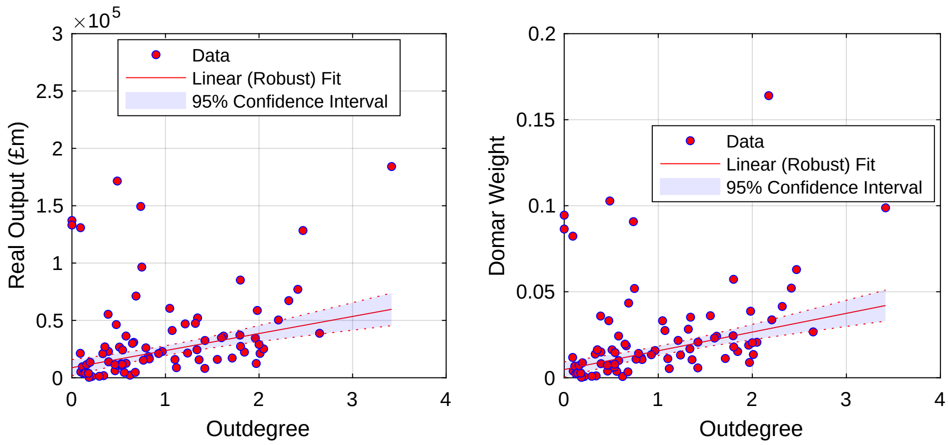
which measures the importance of industry j as a *direct* input supplier to other industries. This measure ranges from 0 if an industry supplies no intermediates to other industries, to N if a single industry is the sole input supplier of every industry in the economy.[6][7]

2.2 Empirical Size-Centrality Relationship

We now present the main empirical finding of this section: a positive relationship between industry size and centrality, both in the long run and over the business cycle.[8]

Long-run relationship. Figure 1 plots the time-averaged *levels* of outdegrees against real output (left panel) and Domar weights (right panel) for all 79 industries. There is a clear positive (and possibly non-linear) relationship, with larger industries on average also tending to be more central. Importantly, since the sum of outdegrees across industries equals $N = 79$ in each year by construction, there is no concern about a spurious relationship driven by, say, a common time trend.

Figure 1. Scatterplot of the average levels of outdegrees and real output and Domar weights, by industry



Notes: All SIC07 2-digit industries are included. Sample covers 1997-2019.

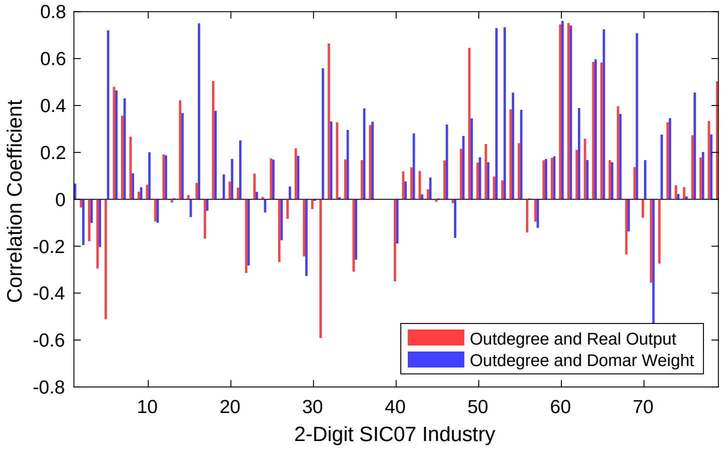
[6]The first-order weighted outdegree is a standard centrality measure in the production networks literature; see, e.g., Acemoglu et al. (2012) and Carvalho (2014). In Appendix A.2, we consider alternative centrality measures, including second-order weighted outdegrees and Bonacich centrality.

[7]While the Domar weight measures the importance of an industry as an *output* supplier to the entire economy, our chosen measure of centrality measures the importance of an industry as an *input* supplier to other industries.

[8]In Appendix A.2, we provide additional results on the UK production network, including the empirical distribution of outdegrees and Bonacich centralities, the stability of input-output linkages, and clustering in the production network.

Business-cycle relationship. Focusing on each industry separately, Figure 2 plots the correlation between the year-on-year *growth rates* of outdegrees and our two size measures. The correlation is positive for 68% of industries when size is measured by real output, and for 76% when measured by Domar weights. In other words, for most industries, periods of growth in size also tend to be periods in which they become more central in the production network.

Figure 2. Correlation between the growth rates of outdegrees and real output and Domar weights, by industry



Notes: Correlations are calculated between year-on-year growth rates. All 2-digit industries are included. Sample covers 1997-2019.

Interpretation. From a modelling perspective, we interpret the relationship in Figure 1 as reflecting the steady-state size-centrality relationship, and that in Figure 2 as reflecting the size-centrality relationship outside of the steady state. Regarding the latter, in a Cobb-Douglas economy—in which all elasticities of substitution are unitary—the input-output network does not change in response to shocks, so industries' centralities are time-invariant. In such an economy, Domar weights are also invariant to shocks. The significant time variation we observe in the data is thus inconsistent with a Cobb-Douglas aggregation of intermediate inputs. This motivates the CES specification we adopt in the model.

In sum, the evidence points to a robust positive size-centrality relationship, both in levels and in growth rates. In Sections 3 and 4, we ask whether standard quantitative models of production networks can generate this relationship, and show that the answer depends critically on the types of shocks present in the economy.

3 Model

In this section, we present a multisector general equilibrium model with an input-output network, which closely follows Atalay (2017). The economy is populated by a representative household and N perfectly competitive industries. The key departure from Atalay (2017) is that we introduce preference (demand) shocks, in addition to technology (supply) shocks. As we show below, this extension is essential for the model to replicate the positive empirical size-centrality relationship documented in Section 2.

This section is organised as follows. We first lay out the baseline model (Section 3.1). We then simplify it to derive analytical results on how industry size and centrality respond to shocks, showing that a model with only technology shocks cannot replicate the empirical size-centrality relationship (Section 3.2). We return to the baseline model to present the log-linearised equilibrium expressions for size and centrality (Section 3.3) and describe our shock-filtering procedure (Section 3.4).

3.1 Baseline Model

3.1.1 Households

The representative household derives utility from the N consumption goods produced by the N competitive industries and disutility from supplying labour. The preferences of the household are given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\log \left[\sum_{i=1}^{N} \left(\frac{D_{it}\xi_i}{\sum_{j=1}^{N} D_{jt}\xi_j} \right)^{\frac{1}{\varepsilon_D}} C_{it}^{\frac{\varepsilon_D - 1}{\varepsilon_D}} \right]^{\frac{\varepsilon_D}{\varepsilon_D - 1}} - \frac{\varepsilon_{LS}}{\varepsilon_{LS}+1} \left(\sum_{i=1}^{N} L_{it} \right)^{\frac{\varepsilon_{LS}+1}{\varepsilon_{LS}}} \right],$$

where β is the subjective discount factor, ξ_i is the time-invariant relative importance of good i in aggregate consumption (with $\sum_{i=1}^{N} \xi_i = 1$), D_{it} is a preference shock to good i in period t (with $D_{it} = 1$ for all i in steady state), C_{it} is the final consumption of good i in period t, and L_{it} is the labour supply to industry i in period t. The parameter ε_D governs how easily the household substitutes across consumption goods, and ε_{LS} governs the elasticity of labour supply.

3.1.2 Producers

Industry i produces a quantity Q_{it} of good i in period t using capital (K_{it}), labour (L_{it}), and an intermediate input bundle (M_{it}) according to the following production function:

$$Q_{it} = A_{it}^{\eta_i} \left[(1-\mu_i)^{\frac{1}{\varepsilon_Q}} \left(\left(\frac{K_{it}}{\alpha_i} \right)^{\alpha_i} \left(\frac{L_{it}}{1-\alpha_i} \right)^{1-\alpha_i} \right)^{\frac{\varepsilon_Q - 1}{\varepsilon_Q}} + \mu_i^{\frac{1}{\varepsilon_Q}} M_{it}^{\frac{\varepsilon_Q - 1}{\varepsilon_Q}} \right]^{\eta_i \frac{\varepsilon_Q}{\varepsilon_Q - 1}},$$

where A_{it} is the factor-neutral technology level of industry i in period t, μ_i and α_i reflect long-run averages in each industry's usage of intermediate inputs, labour, and capital (to be inferred from factor cost shares), ε_Q is the elasticity of substitution between value-added and intermediates, and η_i parametrises returns to scale ($\eta_i = 1$ for constant returns to scale and $0 < \eta_i < 1$ for decreasing returns to scale).

The evolution of capital in each industry is given by

$$K_{i,t+1} = (1-\delta_K) K_{it} + X_{it}.$$

The capital stock is accumulated via an industry-specific bundle of investment goods, X_{it}, and depreciates at a rate δ_K, common across industries. The industry-specific investment bundle is produced by combining the investment goods of potentially all industries:

$$X_{it} = \left(\sum_{j=1}^{N} \left(\Gamma_{ij}^{X} \right)^{\frac{1}{\varepsilon_X}} \left(X_{ijt} \right)^{\frac{\varepsilon_X - 1}{\varepsilon_X}} \right)^{\frac{\varepsilon_X}{\varepsilon_X - 1}},$$

where Γ_{ij}^{X} governs the importance of industry j as an investment-good supplier to industry i, X_{ijt} is the quantity of investment good j used by industry i, and ε_X is the elasticity of substitution across investment goods.

The intermediate input bundle of industry i is defined analogously:

$$M_{it} = \left(\sum_{j=1}^{N} \left(\Gamma_{ij}^{M} \right)^{\frac{1}{\varepsilon_M}} \left(M_{ijt} \right)^{\frac{\varepsilon_M - 1}{\varepsilon_M}} \right)^{\frac{\varepsilon_M}{\varepsilon_M - 1}}, \tag{3}$$

where Γ_{ij}^{M} governs the importance of industry j as an intermediate-good supplier to industry i, M_{ijt} is the quantity of good j used in the production of good i, and ε_M is the elasticity of substitution across intermediate inputs. We say that intermediate inputs are gross complements if $\varepsilon_M < 1$, gross substitutes if $\varepsilon_M > 1$, and Cobb-Douglas if $\varepsilon_M = 1$. As we show below, ε_M is the key parameter determining the size-centrality relationship.

Finally, the market-clearing condition for good j requires that output is allocated to consumption, intermediate use, or investment:

$$Q_{jt} = C_{jt} + \sum_{i=1}^{N}(M_{ijt} + X_{ijt}). \tag{4}$$

3.1.3 Exogenous Processes

Let $A_t \coloneqq (A_{1t}, ..., A_{Nt})^\top$ and $D_t \coloneqq (D_{1t}, ..., D_{Nt})^\top$ denote the vectors of technology and preference shocks, respectively. We assume that the evolution of A_t and D_t follows a geometric random walk:

$$\begin{aligned} \log A_t &= \log A_{t-1} + \omega_t^A, \\ \log D_t &= \log D_{t-1} + \omega_t^D, \end{aligned}$$

where we impose no restrictions on the variance-covariance matrices.

3.2 Simplified Model

Before solving the benchmark model, we build intuition on how industry size and centrality respond to shocks in a simplified version of the baseline model. The aim is to identify the conditions under which the model can replicate the positive empirical size-centrality relationship, documented in Figure 2.

To this end, we simplify the baseline model in two ways. First, the discount factor (β), the elasticity of substitution of demand (ε_D), and the elasticity of substitution between value-added and intermediates (ε_Q) are all assumed to equal one. Second, we assume away capital for each industry by setting $\alpha_i = 0$ for all i. These assumptions reduce the model to a static economy, though it retains a non-trivial temporal dimension as the endogenous variables respond to varying shocks over time.[9]

Under these assumptions, the preferences of the household simplify to

$$U(C_{1t}, ..., C_{Nt}, L_t) = \log \prod_{i=1}^{N} C_{it}^{\gamma_{it}} - \frac{\varepsilon_{LS}}{\varepsilon_{LS} + 1} L_t^{\frac{\varepsilon_{LS}+1}{\varepsilon_{LS}}}, \tag{5}$$

where $\gamma_{it} \coloneqq D_{it}\xi_i/(\sum_{j=1}^{N} D_{jt}\xi_j)$ and $L_t \coloneqq \sum_{i=1}^{N} L_{it}$. The budget constraints are given by

$$\sum_{i=1}^{N} P_{it}C_{it} = W_t L_t + \sum_{i=1}^{N} \Pi_{it},$$

[9]We retain the time subscript to reflect that repeatedly observing this economy under different shock realisations generates time variation in the endogenous variables.

where Π_{it} is the profit from industry i. Each industry's production function simplifies to

$$Q_{it} = \left(A_{it} L_{it}^{1-\mu_i} M_{it}^{\mu_i}\right)^{\eta_i},$$

where M_{it} retains the CES form in equation (3). Note that all elasticities are set to unity *except* ε_M. This is deliberate: as we show below, ε_M is the key parameter determining how shocks affect the size-centrality relationship. When $\varepsilon_M = 1$, the intermediate input bundle reduces to Cobb-Douglas, and expenditure shares on individual inputs are invariant to shocks. When $\varepsilon_M \neq 1$, these shares—and hence the input-output matrix $\mathbf{W}_t$—respond endogenously.

3.2.1 Equilibrium Conditions

We provide selected equilibrium conditions here; the full characterisation of competitive equilibrium is in Appendix B. Cost minimisation by industries yields the conditional factor demands for intermediate input j and the intermediate input bundle:

$$M_{ijt} = \frac{\zeta_i \eta_i \mu_i Q_{it}^{\frac{1}{\eta_i}} W_t^{1-\mu_i} \left(P_{it}^M\right)^{\mu_i - 1 + \varepsilon_M} \Gamma_{ij}^M}{A_{it} P_{jt}^{\varepsilon_M}}, \tag{6}$$

$$M_{it} = \frac{\zeta_i \eta_i \mu_i Q_{it}^{\frac{1}{\eta_i}} W_t^{1-\mu_i} \left(P_{it}^M\right)^{\mu_i - 1}}{A_{it}}, \tag{7}$$

where $\zeta_i := [\eta_i \mu_i]^{-\mu_i} [\eta_i (1-\mu_i)]^{-(1-\mu_i)}$ and P_{it}^M is the ideal price index associated with industry i's intermediate input bundle M_{it}:

$$P_{it}^M = \left[\sum_{j=1}^{N} \Gamma_{ij}^M P_{jt}^{1-\varepsilon_M}\right]^{\frac{1}{1-\varepsilon_M}}.$$

We normalise the consumer price index, P_t, to one. Next, we derive our measures of industry centrality and size in this economy.

3.2.2 Industry Centrality

We measure industry centrality using its first-order (weighted) outdegree, as defined in (2).[10] Combining the definitions in equations (1) and (2) with the optimality conditions (6) and (7), the first-order outdegree of industry j can be expressed as

$$D_{jt}^{\text{out}} = \sum_{i=1}^{N} \omega_{ijt} = \sum_{i=1}^{N} \frac{P_{jt} M_{ijt}}{P_{it}^M M_{it}} = \sum_{i=1}^{N} \Gamma_{ij}^M \left(\frac{P_{jt}}{P_{it}^M}\right)^{1-\varepsilon_M}. \tag{8}$$

[10] In Appendix B.3, we show that the conclusions of this section extend to other commonly used centrality measures, including second-order weighted outdegrees.

The last expression reveals that industry j's centrality depends on prices and the parameter ε_M. In particular, D_{jt}^{out} is strictly increasing in industry j's own price P_{jt} when $\varepsilon_M < 1$ (gross complements), strictly decreasing in P_{jt} when $\varepsilon_M > 1$ (gross substitutes), and independent of P_{jt} when $\varepsilon_M = 1$ (Cobb-Douglas). Since price changes in this economy are driven by technology and preference shocks, equation (8) links the response of centrality to the underlying shocks.

To build intuition, decompose the (i, j)-element of the input-output matrix $\mathbf{W}_t$ as

$$\omega_{ijt} = \left[\frac{P_{jt}}{P_{it}^M}\right]\left[\frac{M_{ijt}}{M_{it}}\right] = \underbrace{\left[\frac{P_{jt}}{P_{it}^M}\right]}_{\text{price effect}}\underbrace{\left[\left(\frac{P_{it}^M}{P_{jt}}\right)^{\varepsilon_M}\Gamma_{ij}^M\right]}_{\text{quantity effect}}. \tag{9}$$

Suppose that industry j is hit by the positive technology shock that reduces the marginal cost of industry j and thus lowers the price of good j. Since good j is only one component of industry i's input bundle, the bundle price P_{it}^M falls by less than P_{jt}, so the relative price P_{jt}/P_{it}^M declines—this is the price effect. At the same time, all industries want to source more from the now-cheaper industry j—this is the quantity effect. If $\varepsilon_M = 1$, the two effects exactly offset, and ω_{ijt} is unchanged. If $\varepsilon_M < 1$, the quantity response is muted because industries cannot easily substitute across inputs, so the price effect dominates: ω_{ijt} falls, and so does industry j's centrality. If $\varepsilon_M > 1$, the quantity effect dominates, and centrality rises.

Now, consider a positive preference shock to good j that shifts up the demand curve and thus increases the price of good j.[11] By the same logic, ω_{ijt} rises if $\varepsilon_M < 1$ and falls if $\varepsilon_M > 1$, so the relationship between the preference shock and centrality is *opposite* to that between the technology shock and centrality.

In Section 4.1, we estimate ε_M and find that the empirically relevant case is $\varepsilon_M < 1$. In this case, a positive technology shock *decreases* industry j's centrality, while a positive preference shock *increases* it.

3.2.3 Industry Size in the Simplified Model

We consider two measures of industry size: real gross output (Q_{jt}) and the Domar weight (λ_{jt}), defined as the ratio of industry j's nominal gross output ($P_{jt}Q_{jt}$) to nominal GDP. We consider both because, as we show below, prices play a key role in the size-centrality relationship, and the two measures respond differently to shocks.

Real output. Real output Q_{jt} increases in response to a positive technology shock (rightward shift in supply) and in response to a positive preference shock (rightward shift

[11] In this static model, preference shocks affect relative prices only under decreasing returns to scale (i.e., $\eta_i < 1$). Under constant returns to scale, relative prices are entirely independent of the demand side. This is not the case in the dynamic baseline model—under constant returns to scale, preference shocks can affect relative prices (and thus industry centrality).

in demand, provided $\gamma_{jt} > 0$). Thus, both types of shocks move real output in the same direction.[12]

Domar weights. In equilibrium, the Domar weight of industry j is given by

$$\lambda_{jt} = \frac{P_{jt}Q_{jt}}{\sum_{i=1}^{N} P_{it}C_{it}} = \frac{P_{jt}C_{jt}}{\sum_{i=1}^{N} P_{it}C_{it}} + \frac{\sum_{i=1}^{N} P_{jt}M_{ijt}}{\sum_{i=1}^{N} P_{it}C_{it}}, \tag{10}$$

where the first equality follows from the expenditure approach to nominal GDP, and the second equality follows from the goods market-clearing condition (4). This equation can be expressed as[13]

$$\lambda_{jt} = \gamma_{jt} + \sum_{i=1}^{N} \eta_i \mu_i \omega_{ijt} \lambda_{it}.$$

Assuming $\eta_i = \eta$ and $\mu_i = \mu$ for all i for simplicity, we can write this in matrix form and solve for the Domar weights:

$$\lambda_t = \left[\mathbf{I}_N - \eta\mu\mathbf{W}_t^\top\right]^{-1} \gamma_t = \left[\mathbf{I}_N + \eta\mu\mathbf{W}_t^\top + (\eta\mu)^2 \left(\mathbf{W}_t^\top\right)^2 + ...\right] \gamma_t, \tag{11}$$

where $\lambda_t := (\lambda_{1t}, \dots, \lambda_{Nt})^\top$, $\gamma_t := (\gamma_{1t}, \dots, \gamma_{Nt})^\top$, $\mathbf{I}_N$ is the $N \times N$ identity matrix, the matrix $[\mathbf{I} - \eta\mu\mathbf{W}_t]^{-1}$ is the Leontief inverse (more precisely, its transpose),[14] and the second equality uses the power series expansion. Equation (11) shows that the Domar weights depend on two objects: preference shares and the corresponding column of the Leontief inverse. Intuitively, on top of the sales that an industry generates by selling final goods to the representative household, its total sales in equilibrium also include the sale of intermediate goods which ultimately get transformed into final goods sold to the household.[15]

Now consider a positive technology shock to industry j, which lowers P_{jt}. As argued in Section 3.2.2, ω_{ijt} falls when $\varepsilon_M < 1$. This then implies that the entries in the j-th row

[12] While any industry's real output and Domar weight may respond to shocks originating in other industries, the majority of variation in an industry's size and centrality tends to be driven by its own shocks. We demonstrate this in Section 4 using the calibrated baseline model.

[13] To see this, the first term on the right-hand side of equation (10) is the expenditure share on good j and thus equals γ_{jt} under Cobb-Douglas preferences (5). The second term on the right-hand side can be rewritten as

$$\frac{\sum_{i=1}^{N} P_{jt}M_{ijt}}{\sum_{i=1}^{N} P_{it}C_{it}} = \sum_{i=1}^{N} \frac{P_{jt}M_{ijt}}{P_{it}^M M_{it}} \frac{P_{it}^M M_{it}}{P_{it}Q_{it}} \frac{P_{it}Q_{it}}{\sum_{i=1}^{N} P_{it}C_{it}} = \sum_{i=1}^{N} \omega_{ijt}\eta_i\mu_i\lambda_{it},$$

where the last equation follows from the definition of ω_{ijt}, industry i's expenditure share on the intermediate input bundle, and the definition of the Domar weight.

[14] This matrix records the direct and indirect exposures through the production network. In particular, the (j, i) element of the Leontief inverse measures the importance of industry i as a direct and indirect input supplier to industry j in the economy.

[15] In the extreme case where the production network contains no non-trivial linkages ($\mathbf{W}_t = \mathbf{I}_N$), the Domar weights simplify to $\lambda_t = [1 + \eta\mu/(1 - \eta\mu)]\, \gamma_t = (1 - \eta\mu)^{-1}\gamma_t$.

Table 1: Effect of Technology and Preference Shocks on Industry j's Centrality and Size

Variable	Positive Technology Shock ($A_{jt} \uparrow$)	Positive Preference Shock ($D_{jt} \uparrow$)
Industry centrality (D^{out}_{jt})	$\downarrow$	$\uparrow$
Industry size: Domar weight (λ_{jt})	$\downarrow$	$\uparrow$
Industry size: real output (Q_{jt})	$\uparrow$	$\uparrow$

Notes: The effects shown assume $\varepsilon_M < 1$ (gross complementarity across intermediate inputs). The signs reverse for negative shocks. See Sections 3.2.2–3.2.4 for derivations.

of the matrix $\mathbf{W}_t^\top$ fall. Therefore, equation (11) implies that λ_{jt} falls, since the positive technology shock will leave γ_t unchanged. Conversely, a positive preference shock to industry j raises the j-th element of γ_t (and lowers the others, since $\sum_{i=1}^{N} \gamma_{it} = 1$) and, when $\varepsilon_M < 1$, raises ω_{ijt}. Both channels push λ_{jt} upward.

Therefore, for the empirically relevant case where $\varepsilon_M < 1$, there is a negative relationship between the technology shock and Domar weights, while there is a positive relationship between the preference shock and Domar weights.

3.2.4 Summary

Table 1 collects the results from the preceding analysis for the empirically relevant case $\varepsilon_M < 1$. Technology shocks move centrality and real output in *opposite* directions: a positive technology shock raises output but lowers centrality. By contrast, preference shocks move all three variables—centrality, Domar weights, and real output—in the *same* direction.[16]

The key implication is that models featuring *only* technology shocks—as is standard in the quantitative production networks literature (e.g., Atalay, 2017)—imply a strongly negative relationship between real output and centrality outside of the steady state. This is inconsistent with the positive empirical relationship documented in Figure 2. Introducing demand-type shocks can reconcile the model with the data. We confirm this finding quantitatively in Section 4.3.

3.3 Equilibrium in the Baseline Model

We now return to the baseline model of Section 3.1 and present its solution. We characterise the non-stochastic steady state of the baseline model, log-linearise around that steady state, and solve for the policy functions. All derivations are provided in Appendix C.

[16]Technology shocks may imply a positive size-centrality relationship for industries *other than* the one in which the shock originated. In practice, the majority of variation in industries' size and centrality is driven by their own shocks, hence our emphasis on the shock-originating industry.

The evolution of output can be written as

$$\Delta \log Q_{t+1} = \mathbf{\Pi}_1 \Delta \log Q_t + \begin{bmatrix} \mathbf{\Pi}_2 \vdots \mathbf{\Pi}_4 \end{bmatrix} \begin{bmatrix} \omega_t^A \\ \omega_t^D \end{bmatrix} + \begin{bmatrix} \mathbf{\Pi}_3 \vdots \mathbf{\Pi}_5 \end{bmatrix} \begin{bmatrix} \omega_{t-1}^A \\ \omega_{t-1}^D \end{bmatrix}, \tag{12}$$

where $Q_t := (Q_{1t}, ..., Q_{Nt})^\top$, ω_t^A and ω_t^D are the vectors of exogenous technology and preference shocks, respectively, and the $N \times N$ matrices $\mathbf{\Pi}_1$, $\mathbf{\Pi}_2$, $\mathbf{\Pi}_3$, $\mathbf{\Pi}_4$, and $\mathbf{\Pi}_5$ are functions of the model parameters only. We solve for these matrices in Appendix C. The baseline approach in Atalay (2017) assumes away preference shocks (i.e., $\omega_t^D = 0$) and uses data on industries' output growth to filter out the technology shocks using equation (12). As in Foerster, Sarte, and Watson (2011), the equilibrium in this model permits a VARMA(1,1) representation in industries' output growth rates if preference shocks are assumed away. In contrast to them, we allow for preference shocks. Thus, to filter all $2N$ shocks in the model, we instead use data on industries' value-added and prices, which then admit a VARMA(1,1) representation. We present this result in Section 3.4.

3.3.1 Industry Size

We continue to use real output and Domar weights of industries as measures of industry size. Let $\hat{q}_t$, $\hat{\lambda}_t$, and $\hat{p}_t$ denote vectors of log-deviations from steady state of real output, Domar weights, and prices, respectively. Log-deviations of nominal gross output are $\hat{n}_t := \hat{q}_t + \hat{p}_t$, and Domar weights are given by $\hat{\lambda}_t = \hat{p}_t + \hat{q}_t - \widehat{ngdp}_t \cdot \iota$ where $\widehat{ngdp}_t$ is the log-deviation of nominal GDP and ι is an $N \times 1$ vector of ones.

The two measures of industry size can be expressed as functions of capital and shocks:

$$\begin{bmatrix} \Delta \hat{q}_t \\ \Delta \hat{\lambda}_t \end{bmatrix} = \begin{bmatrix} \mathbf{\Phi}_k \\ \mathbf{\Lambda}_k \end{bmatrix} \Delta \hat{k}_t + \begin{bmatrix} \mathbf{\Phi}_a \\ \mathbf{\Lambda}_a \end{bmatrix} \omega_t^A + \begin{bmatrix} \mathbf{\Phi}_d \\ \mathbf{\Lambda}_d \end{bmatrix} \omega_t^D, \tag{13}$$

where all matrices in square brackets are functions of model parameters only.

3.3.2 Industry Centrality

As in equation (8) in the simplified model, the first-order outdegrees can be expressed as a function of prices: $\hat{d}_t^{\text{out}} = \mathbf{D}\hat{p}_t$, where $\mathbf{D}$ depends on model parameters. The log-deviation of first-order outdegrees from their steady-state values can be expressed as

$$\Delta \hat{d}_t^{\text{out}} = \mathbf{D}_k \Delta \hat{k}_t + \mathbf{D}_a \omega_t^A + \mathbf{D}_d \omega_t^D, \tag{14}$$

where the matrices $\mathbf{D}_k$, $\mathbf{D}_a$, and $\mathbf{D}_d$ are functions of model parameters only. Together, equations (13) and (14) allow us to compute the model-implied size and centrality once the shocks $\{\omega_t^A, \omega_t^D\}$ have been filtered.

3.4 Model Filter in the Baseline Model

We now describe the procedure for recovering the full set of technology and preference shocks from data. Denote by $\hat{v}_t$ the vector of log-deviations of value-added from the steady state. In our baseline calibration, we set $\varepsilon_Q = 1$ (see Section 4.2), which implies $\hat{v}_t = \hat{q}_t$ (Appendix C.3). Thus, by matching value-added data, we simultaneously match industries' real output. We target value-added since we do not observe output growth by industry at a quarterly frequency. Since outdegrees depend only on prices (see equation (14)), matching price data ensures that the model also matches industries' centrality.

Appendix C.3 shows that our model filter follows a VARMA(1,1) process:

$$\begin{bmatrix} \Delta\hat{v}_{t+1} \\ \Delta\hat{p}_{t+1} \end{bmatrix} = \begin{bmatrix} \tilde{\mathbf{V}}_v & \mathbf{0} \\ \mathbf{0} & \tilde{\mathbf{P}}_p \end{bmatrix} \begin{bmatrix} \Delta\hat{v}_{t} \\ \Delta\hat{p}_{t} \end{bmatrix} + \begin{bmatrix} \mathbf{V}_a & \mathbf{V}_d \\ \mathbf{P}_a & \mathbf{P}_d \end{bmatrix} \begin{bmatrix} \omega^A_{t+1} \\ \omega^D_{t+1} \end{bmatrix} + \begin{bmatrix} \tilde{\mathbf{V}}_a & \tilde{\mathbf{V}}_d \\ \tilde{\mathbf{P}}_a & \tilde{\mathbf{P}}_d \end{bmatrix} \begin{bmatrix} \omega^A_{t} \\ \omega^D_{t} \end{bmatrix},$$

where all matrices only depend on the model parameters in Table 3. Solving for the shocks yields

$$\begin{bmatrix} \omega^A_{t+1} \\ \omega^D_{t+1} \end{bmatrix} = \begin{bmatrix} \mathbf{V}_a & \mathbf{V}_d \\ \mathbf{P}_a & \mathbf{P}_d \end{bmatrix}^{-1} \begin{bmatrix} \Delta\hat{v}_{t+1} \\ \Delta\hat{p}_{t+1} \end{bmatrix} - \begin{bmatrix} \mathbf{V}_a & \mathbf{V}_d \\ \mathbf{P}_a & \mathbf{P}_d \end{bmatrix}^{-1} \begin{bmatrix} \tilde{\mathbf{V}}_v & \mathbf{0} \\ \mathbf{0} & \tilde{\mathbf{P}}_p \end{bmatrix} \begin{bmatrix} \Delta\hat{v}_{t} \\ \Delta\hat{p}_{t} \end{bmatrix} - \begin{bmatrix} \mathbf{V}_a & \mathbf{V}_d \\ \mathbf{P}_a & \mathbf{P}_d \end{bmatrix}^{-1} \begin{bmatrix} \tilde{\mathbf{V}}_a & \tilde{\mathbf{V}}_d \\ \tilde{\mathbf{P}}_a & \tilde{\mathbf{P}}_d \end{bmatrix} \begin{bmatrix} \omega^A_{t} \\ \omega^D_{t} \end{bmatrix}. \tag{15}$$

With some initial conditions for the technology and preference shocks (e.g., $\omega^A_t = \omega^D_t = 0$), we iterate equation (15) forward to recover the entire sequence of shocks.[17][18]

Once we have filtered the shocks, we compute the model-implied size and centrality variables using equations (13) and (14).

4 Quantitative Analysis

We now take the model to the data. We first estimate the key parameter ε_M and calibrate the remaining parameters (Sections 4.1–4.2). We then filter out technology and preference shocks and examine the model-implied size-centrality relationship (Section 4.3). The main result of this section is that the quantitative analysis confirms the analytical findings of the simplified model: a model with both technology and preference shocks can replicate the positive empirical size-centrality relationship, while a model with only technology shocks cannot.

[17] We set the burn-in period to 7 quarters, discarding the first 7 filtered values. The Kalman filter yields equivalent results. Convergence requires the eigenvalues of the relevant companion matrix to be less than 1 in modulus; see Appendix C for details.

[18] Note that we do not require data on capital ($\hat{k}_t$). As shown by Atalay (2017), after solving for a variable in terms of capital and the shocks, the first-order log-linear approximation to the equilibrium dynamics around the steady state has a VARMA representation, as long as an invertibility condition is satisfied.

4.1 Estimating the Elasticity of Substitution across Intermediates

The key parameter is ε_M, the elasticity of substitution across intermediate inputs. As shown in Section 3.2, ε_M governs how industry size and centrality respond to technology and preference shocks. Moreover, as Atalay (2017) shows, this parameter will directly affect the relative importance of industry-specific versus common shocks: when ε_M is low (high), industry-specific (common) shocks will tend to be more important in explaining aggregate GDP dynamics.

We estimate ε_M, following the procedure proposed by Atalay (2017). The cost-minimisation conditions of industry i, summarised by equation (9), yield the following estimating equation:

$$\Delta \log \left(\frac{P_{jt} M_{ijt}}{P_{it}^M M_{it}} \right) = \phi_t + (1 - \varepsilon_M) \Delta \log \left(\frac{P_{jt}}{P_{it}^M} \right) + \nu_{ijt}. \tag{16}$$

Since relative prices on the right-hand side are endogenous, OLS is inconsistent. We instrument using industries' heterogeneous (direct and indirect) exposures to UK military spending, which is plausibly exogenous to the regression error ν_{ijt}.

Table 2: Estimation results for equation (16)

Second stage	(1) OLS	(2) OLS, Year FE	(3) IV	(4) IV, Year FE
ε_M	0.680***	0.680***	0.273***	0.345**
	(0.015)	(0.015)	(0.266)	(0.263)
First stage: Dependent variable is $\Delta \log(P_{jt}/P_{it}^M)$.				
military spending shock$_{it}$			0.331***	0.385***
			(0.033)	(0.050)
military spending shock$_{jt}$			-0.205***	-0.185***
			(0.029)	(0.032)
military spending shock$_{jt\text{'s suppliers}}$			-0.051***	-0.045***
			(0.011)	(0.012)
N	35343	35343	34623	34623
Adjusted R^2	0.013	0.014	.	.
Wu-Hausman test p-value			0.124	0.204
Cragg-Donald Statistic			37.968	38.593
Year Fixed Effects	No	Yes	No	Yes

Notes: Statistical significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 2 reports the results. OLS yields estimates of ε_M around 0.68, but these estimates are inconsistent due to the endogeneity of P_{jt}/P_{it}^M. IV estimation—which uses three instruments based on industries' heterogeneous exposures to UK military spending—yields estimates of ε_M between 0.27 and 0.35, depending on whether year fixed effects are included. All first-stage instruments are significant at the 1% level, and the second-stage estimate is significant at the 5% level in our preferred specification (column 4, with year fixed effects). We set ε_M equal to 0.35 in the baseline calibration. Importantly, all IV estimates satisfy $\varepsilon_M < 1$, confirming the gross complementarity assumption maintained throughout Section 3.2.

4.2 Calibration

The remaining elasticity parameters are calibrated following Atalay (2017).[19] We assume constant returns to scale ($\eta_i = 1$) and calibrate the industry-specific parameters α_i, ξ_i, μ_i, Γ^M_{ij}, and Γ^X_{ij} using UK supply and use tables, averaging over 1997–2019. The quarterly depreciation rate is set to 1.4%, consistent with an annual rate of roughly 5.5%, as estimated by Oulton and Wallis (2016).[20] Table 3 summarises all calibrated parameter values.

Table 3. Baseline calibration of model parameters

Parameter(s)	Value	Source
ε_Q	1	Atalay (2017)
ε_M	0.35	Estimated
ε_X	1	Atalay (2017)
ε_D	1	Atalay (2017)
ε_{LS}	2	Atalay (2017)
η_i	1	Atalay (2017)
α_i	Average share of labour expenses in i's GVA	UK's Supply and Use Tables
ξ_i	Average i's share of final demand	UK's Supply and Use Tables
μ_i	Average share of intermediates in i's GVA	UK's Supply and Use Tables
δ_K	0.014	Based on Oulton and Wallis (2016)
Γ^M_{ij}	Average share of intermediate inputs from j to i	UK's Supply and Use Tables
Γ^X_{ij}	Average share of GFCF flows from j to i	UK's Supply and Use Tables
β	0.99	Atalay, Drautzburg, and Wang (2018)

Notes: 'Averages' refer to average values over 1997-2019. i and j denote 2-digit industries.

4.3 Model-Implied Size-Centrality Relationship

We now examine whether the calibrated model can replicate the empirical size-centrality relationship. We filter technology and preference shocks using equation (15), and then compute size and centrality using equations (13) and (14).

Panel (a) of Figure 3 shows the industry-by-industry correlations between our measures of size and centrality implied by the filtered shocks. The broad pattern is mostly positive,

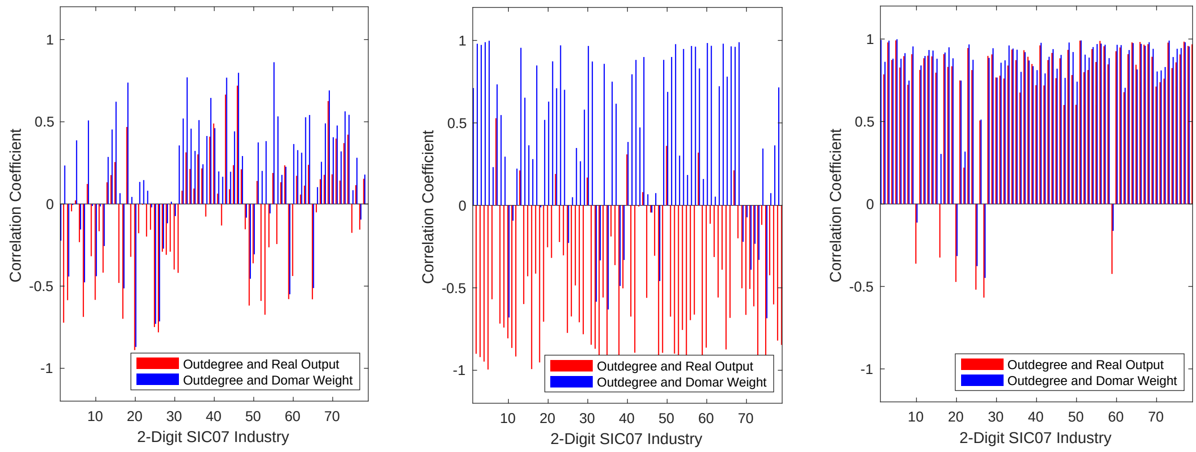
[19] Several recent papers estimate or calibrate ε_Q to be somewhat below 1: for example, Peter and Ruane (2023) estimate 0.6 for the US, and Baqaee and Farhi (2019) use 0.5. Our results are robust to $\varepsilon_Q = 0.5$ and to reasonable alternative values of ε_D (see Appendix D).

[20] We assume a common depreciation rate across sectors. In practice, asset composition varies across sectors and over time, but our findings are not sensitive to reasonable variations in δ_K.

mirroring the empirical counterpart in Figure 2. The remaining discrepancies arise mainly because (i) the model-implied relationship $\hat{d}_t^{\text{out}} = \mathbf{D}\hat{p}_t$ does not hold exactly in the data, and (ii) value-added and gross output growth are not identical in the data (unlike in the model, where $\varepsilon_Q = 1$ implies $\hat{v}_t = \hat{q}_t$).

Figure 3. Model-implied relationship between size and centrality

(a) All shocks (b) Technology shocks (c) Preference shocks



To isolate the role of each type of shock, panels (b) and (c) of Figure 3 display the size-centrality relationship generated by technology shocks alone and preference shocks alone, respectively. Consistent with the analytical predictions of the simplified model (Table 1), technology shocks induce a positive correlation between Domar weights and outdegrees, but a *negative* correlation between real output and outdegrees. Preference shocks, by contrast, induce a strong positive correlation between *either* measure of size and centrality.

These results have a clear implication: to match the empirical size-centrality relationship in Figure 2, a combination of technology and preference shocks is needed. A model featuring only technology shocks would produce a counterfactually strong negative comovement between outdegrees and real output outside the steady state. The relative importance of each type of shocks—which depends partly on the filtering targets—will determine the extent to which the model can match the empirical size-centrality relationship.[21]

Finally, recall from Figure 1 that the empirical size-centrality relationship in the steady state is positive. This is also the case in this model, given our calibration approach. The steady-state input-output shares are calibrated to match the average empirical values over 1997–2019, which underpin the cross-sectional patterns in Figure 1. The model therefore matches the empirical size-centrality relationship both in and out of steady state.[22]

[21] In Appendix C.8, we show that industries' own shocks account for the bulk of variation in own size and centrality, reinforcing the interpretation of the results in Figure 3.

[22] Appendix D presents robustness checks for key parameters of the model.

5 Application to UK Productivity Slowdown

The framework developed in Sections 3–4 recovers the full set of technology and preference shocks from data on UK industries, while accounting for the role of the production network in transmitting these shocks across industries. A natural application is to use this framework to analyse the UK's productivity growth slowdown following the 2008–09 recession. Relative to existing accounting decompositions of the slowdown (e.g., Tenreyro, 2018), our approach offers two advantages: (i) it identifies the underlying shocks—both idiosyncratic and common—driving the slowdown, rather than simply attributing it to industries; and (ii) it accounts for the fact that, through the production network, a shock to one industry can affect aggregate productivity via its impact on other industries, with the magnitude of this transmission depending on the shocked industry's size and centrality.[23]

We proceed as follows. We first set up the decomposition of aggregate labour productivity growth in terms of industries and shocks. We then present the model fit, the accounting contributions of industries, and the contributions of idiosyncratic and common shocks. Finally, we briefly examine the Covid-19 period.

5.1 Decomposing Aggregate Productivity Growth

Defining aggregate labour productivity as aggregate value-added divided by aggregate labour, we can write it in log-linear form as

$$\hat{Y}_t - \hat{L}_t = \sum_{i=1}^{N} \left(S_i^Y \hat{v}_{it} - L_i \hat{l}_{it} \right) = S^Y \cdot \hat{v}_t - L \cdot \hat{l}_t,$$

where S_i^Y is the steady-state share of industry i's value-added in aggregate value-added:

$$S_i^Y = \frac{(1-\mu_i) P_i Q_i}{\sum_{j=1}^{N} (1-\mu_j) P_j Q_j},$$

and L_i denotes industry i's steady-state share of aggregate labour. The growth of aggregate labour productivity is thus given by

$$\begin{aligned} \Delta\hat{Y}_t - \Delta\hat{L}_t &= S^Y \Delta\hat{v}_t - L \Delta\hat{l}_t \\ &= S^Y \left(\mathbf{V}_k \Delta\hat{k}_t + \mathbf{V}_a \omega_t^A + \mathbf{V}_d \omega_t^D \right) - L \left(\mathbf{L}_k \Delta\hat{k}_t + \mathbf{L}_a \omega_t^A + \mathbf{L}_d \omega_t^D \right), \end{aligned} \quad (17)$$

where the matrices in bold depend on the model parameters only.[24] Equation (17) permits

[23] See also Dacic and Melolinna (2022) for an earlier version of this work.

[24] See Appendix C for the derivation.

two types of decomposition: by industry (which industry i contributed how much to aggregate productivity growth?) and by shock (which shock—idiosyncratic or common—drove aggregate productivity growth?). The distinction matters because an industry's accounting contribution may reflect shocks originating elsewhere in the network, while an idiosyncratic shock to one industry may propagate to many others.

In addition to idiosyncratic shocks, there may be common shocks that affect all industries. Once we have filtered out the shocks, we perform factor analysis to extract the common component. In particular, we assume that there are up to two common factors affecting industries' technology.[25] We assume away common preference shocks, since, unlike aggregate technology shocks, they lack a clear interpretation. Industry-specific preference shocks, by contrast, can be interpreted as shifts in relative demand across goods.[26]

The relative importance of common versus industry-specific shocks will mainly depend on two factors: (i) the extent to which the targeted variables are correlated across industries (in our case, value-added and labour growth), and (ii) the value of ε_M. If ε_M is close to 0, industries source inputs from each other in an almost complementary fashion, implying they do not substitute across them easily. In that case, a shock to a highly central industry (such as finance) will act much like a common shock since many industries are exposed to it and cannot substitute away. If instead ε_M is close to (or larger than) one, industries can substitute across their input-suppliers relatively more easily, so genuine common shocks will be a much more likely source of cross-correlation in the observed value-added/labour growth than shocks to any particular industry, no matter how central that industry may be.

Since we are considering a first-order approximation around a non-stochastic steady state, aggregate labour productivity depends on industries' value-added and labour through the time-invariant matrices, S^Y and L. In our baseline calibration, the values of the entries in these matrices are related to the average data counterparts over our sample (1997–2019).[27] As a robustness check, we also consider two alternative calibrations: one based on the 1997 data and another based on the 2019 data. These alternative calibrations yield very similar results to our baseline findings (see Figure E.2 in Appendix E).

[25] By construction, the common and sectoral (idiosyncratic) components add up to the total shock for each industry. Note that Atalay (2017), whose dataset contains 30 sectors, allows for a single common component. We have 79 industries, so allowing for an additional common factor seems appropriate. This is also comparable to Foerster, Sarte, and Watson (2011), who allow for two common factors in a model with 117 sectors.

[26] Nonetheless, allowing for common preference shocks, consisting of one or two common factors, has a negligible effect on our results.

[27] An important phenomenon that has taken place since the late 1990s in the UK has been the decline of manufacturing sector's share of labour, up until the 2008-09 crisis (from 16% in 1997 to 9% in 2009). The steady-state matrix of industries' shares of total labour, L, will thus not reflect the declining relative importance of manufacturing for aggregate productivity growth *solely* due to its share of total labour falling.

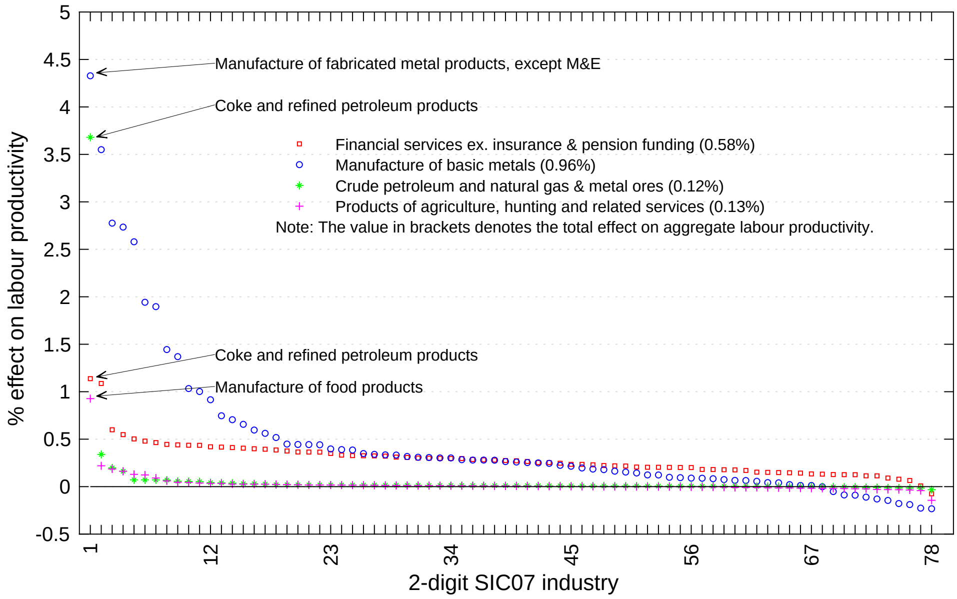
To filter the shocks for this application, we match industries' value-added and labour growth (rather than value-added and prices as in Section 3.4), since capturing the actual movements in industries' labour is essential for decomposing labour productivity. The resulting filter takes the same VARMA(1,1) form:

$$\begin{bmatrix} \Delta \hat{v}_{t+1} \\ \Delta \hat{l}_{t+1} \end{bmatrix} = \begin{bmatrix} \tilde{\mathbf{V}}_v & \mathbf{0} \\ \mathbf{0} & \tilde{\mathbf{L}}_l \end{bmatrix} \begin{bmatrix} \Delta \hat{v}_t \\ \Delta \hat{l}_t \end{bmatrix} + \begin{bmatrix} \mathbf{V}_a & \mathbf{V}_d \\ \mathbf{L}_a & \mathbf{L}_d \end{bmatrix} \begin{bmatrix} \omega_{t+1}^A \\ \omega_{t+1}^D \end{bmatrix} + \begin{bmatrix} \tilde{\mathbf{V}}_a & \tilde{\mathbf{V}}_d \\ \tilde{\mathbf{L}}_a & \tilde{\mathbf{L}}_d \end{bmatrix} \begin{bmatrix} \omega_t^A \\ \omega_t^D \end{bmatrix}. \tag{18}$$

As before, all matrices are functions of the model parameters in Table 3. Solving for the shocks and iterating the resulting equation forward, we can recover the entire sequence of shocks.

Role of size and centrality. The attribution of the productivity growth slowdown to industries in equation (17) will be affected by their size and centrality in two ways. First, for a given $\Delta \hat{v}_t$ or $\Delta \hat{l}_t$, larger industries will affect aggregate labour productivity more to the extent that their share of total value-added, S^Y, and share of total labour, L, is larger. Second, column $j \in N$ in the matrices $\mathbf{V}_i$ ($\mathbf{L}_i$) for $i \in (k, a, d)$, which determine how value-added (labour) in all industries respond to changes in industry j's capital and its technology and preference shocks, will generally have larger entries (in absolute size) for larger and/or more central industries. Figure 4 illustrates this: it shows the effect of a 10% technology shock in selected industries on *other* industries' labour productivity. Shocks to the most central industries in the input-output network (e.g., basic metals and finance) affect a disproportionately large number of other industries relative to shocks to less central industries (e.g., agriculture and mining).

Figure 4. Ordered responses of other industries' labour productivity to a 10% technology shock in the selected four industries



Notes: The x-axis has 78 ticks, corresponding to 78 industries other than the industry which is the source of the technology shock (in total, there are $N = 79$ industries). In each of the four cases, the first tick corresponds to the most affected industry other than the industry in which the shock has originated, the second tick to the second most affected industry other than the industry in which the shock has originated, and so on.

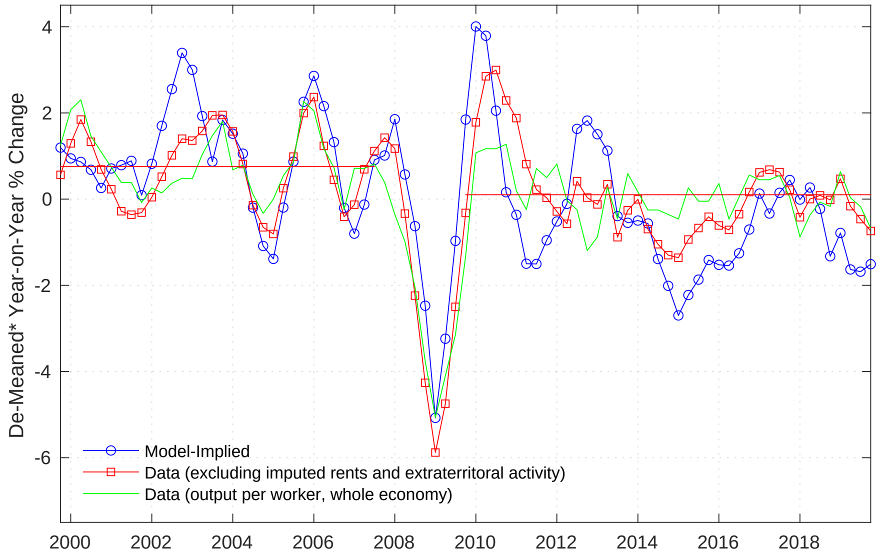
5.2 Results: Model Fit

Using equation (17), we can directly compute the model-implied aggregate labour productivity growth, since the value-added and labour dynamics are exactly matched in the data. Figure 5 compares the resulting path with the data. The correlation coefficient between the model-implied path and the data is 0.8 (0.6) if imputed rents and extraterritorial activity are excluded (included).[28] To quantify the growth puzzle, we compare average post-crisis (2010Q1–2019Q4) and pre-crisis (1999Q1–2007Q4) growth. We find that the model-implied growth puzzle is around 0.26 percentage points (pp) lower growth per quarter post-crisis, somewhat larger but close to the data counterpart of 0.18pp.

Using the filtered shocks from equation (18) and the dynamics of aggregate labour productivity growth given by equation (17), we can go beyond the accounting decomposition of the slowdown into industries' contributions and analyse the importance of various shocks in driving the puzzle. We make this distinction clear below.

[28]We exclude imputed rents and extraterritorial activity (section T) in the model, but we include the ONS's headline measure of output per worker for completeness. We do not exactly match the official ONS aggregate data for two reasons. First, the log-linear approximation of the model dynamics does not exactly match the ONS's sectoral aggregation. Second, the parameters of the matrices S^Y and L are time-invariant over the sample, unlike in the data.

Figure 5. Aggregate labour productivity de-meaned growth: data vs. model



Notes: *The model-implied series is based on de-meaned data on industries' value-added and jobs growth. The two series based on ONS data have been de-meaned so as to ensure that all three series have the same mean over the period shown in the figure (1999:Q4 to 2019:Q4). The two horizontal lines show the pre-2008 and post-2010 averages in the data.

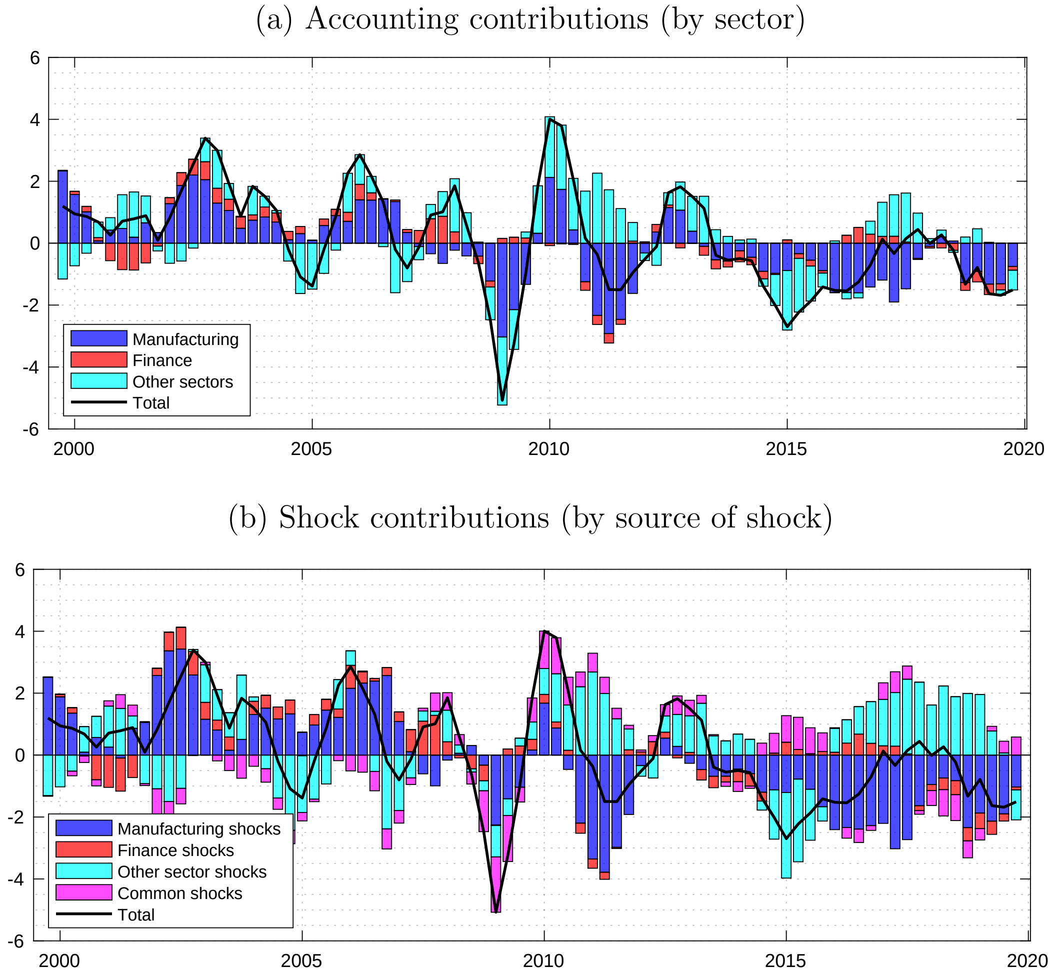
5.3 Accounting Contributions of Industries

We first decompose aggregate labour productivity growth into industries' contributions using equation (17). These contributions reflect all underlying shocks—common or industry-specific—that have impacted a given industry and thereby affected aggregate labour productivity. This exercise is conceptually similar to the decomposition in Tenreyro (2018), who finds that three-quarters of the UK's productivity growth puzzle can be accounted for by manufacturing and finance.[29]

Panel (a) of Figure 6 shows the results of our historical decomposition. The dynamics of aggregate productivity have been significantly driven by the manufacturing sector. Albeit significantly smaller, the contributions from finance are also non-negligible. Note that these contributions reflect all underlying shocks, whether industry-specific or common. The difference between the average pre-crisis and post-crisis contribution to quarterly aggregate productivity growth from manufacturing is -0.37pp. Therefore, the post-2010 slowdown in manufacturing growth alone can account for more than the entirety of the aggregate growth puzzle (-0.26pp). Other sectors that have contributed negatively (albeit to a much lesser extent than manufacturing) include mining, finance, and ICT. In contrast, real estate, along with admin and support, and public services, have provided a partial

[29]The main difference relative to Tenreyro (2018) is that the linearity of aggregate labour productivity in sectoral contributions and/or shocks here comes from the log-linear nature of the approximation around the steady state. Instead, the approach in Tenreyro (2018) directly exploits the bottom-up aggregation of national statistics.

Figure 6. Historical decomposition of contributions to aggregate labour productivity fluctuations



Notes: All 2-digit industries included.

offset given their better productivity performance post-crisis than pre-crisis.

5.4 Contributions of Industry-Specific and Common Shocks

We now decompose aggregate labour productivity growth into contributions from the underlying shocks, including common shocks, using equation (17). This decomposition differs from the accounting decomposition above: the total contribution of an idiosyncratic shock to, say, finance includes its effect on aggregate productivity via *all* industries—not only finance—through the production network. Panel (b) in Figure 6 shows the results.

We can see that the common technology shock has been important around the 2008-9 crisis years. On net, we find that its contributions have been generally more positive post-crisis than pre-crisis. In contrast, the contributions of manufacturing-specific shocks have been persistently negative and very sizeable since the crisis, unlike in the pre-crisis period. It also appears that the manufacturing sector's *accounting* contributions, shown in panel (a), tend to subsume the contributions from common technology shocks to a meaningful degree. Intuitively, although technology shocks in this model propagate both upstream (i.e., towards one's input suppliers) and downstream (i.e., towards one's input purchasers), the downstream propagation is stronger.[30] As the most central input-supplying sector in the economy by far, manufacturing thereby ends up absorbing most of the common technology shock.

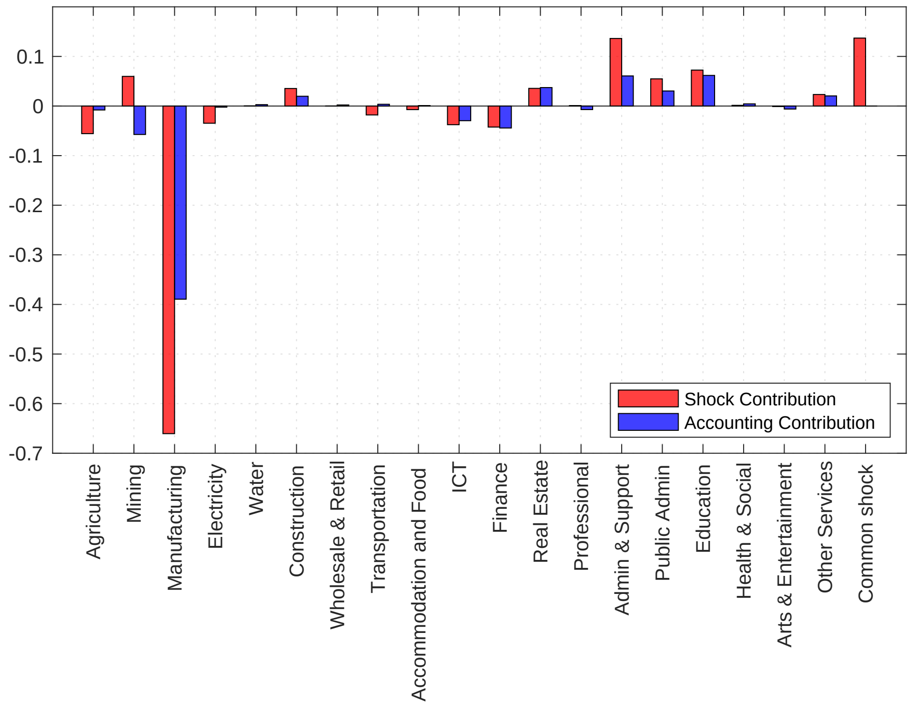
Figure 7 compares the average pre-crisis and post-crisis contributions of each shock source. The red bars show that the drag from more negative manufacturing-specific shocks post-crisis has been particularly large, at -0.65pp per quarter. This has been partially offset by common technology shocks, which have, on average, been 0.13pp higher per quarter post-crisis than pre-crisis. Several sectors—most notably, administrative and support services activities ("Admin & Support") and mining and quarrying ("Mining")—have experienced significantly more positive shocks post-crisis relative to pre-crisis than their accounting contributions (reflecting possibly all shocks) would suggest.[31][32]

[30]Intuitively, technology shocks change the prices faced by purchasers of inputs, creating powerful downstream propagation. See Acemoglu, Akcigit and Kerr (2016) for a more elaborate discussion of the propagation of shocks upstream and downstream in the network. Note that demand-side shocks have much milder effects on prices and propagate mainly upstream as affected industries adjust their production levels and thus input demands.

[31]Another way to visualise the growth puzzle is to compare the performance of sectors post-crisis *over time* (i.e., without averaging) relative to their pre-crisis average performance. Figure E.1 in Appendix E shows that manufacturing is a clear outlier, having underperformed by far the most in its pre-crisis growth, followed by finance, ICT, and mining. Our results suggest fairly persistent deviations from pre-crisis contributions; hence, they echo those obtained by averaging pre- and post-crisis performance in Figure 7.

[32]In Appendix E, we show that our baseline results around the contributions of shocks to the UK's productivity growth puzzle are largely robust to varying ε_M.

Figure 7. Contributions to the growth puzzle: sectors vs. shocks

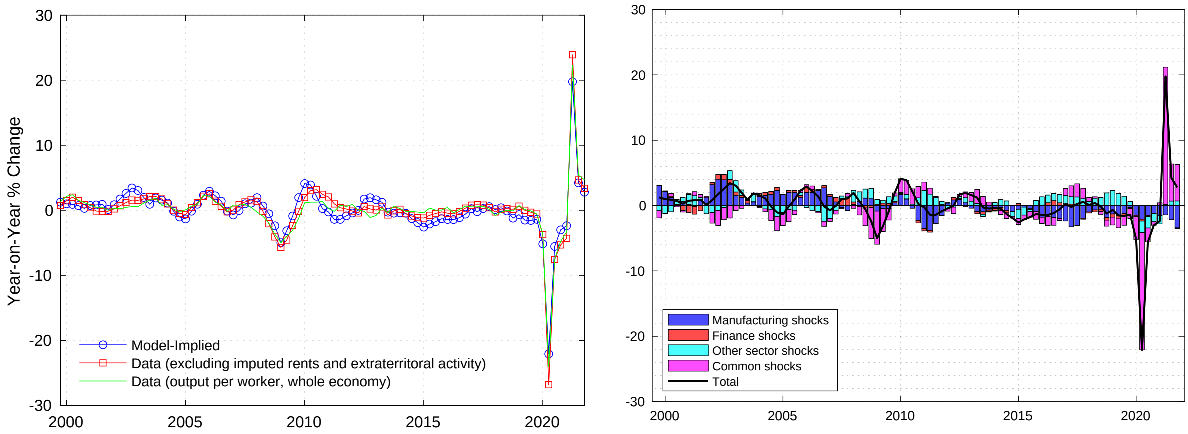


5.5 The Covid-19 Pandemic

Our sample in the preceding analysis ended in 2019Q4, just before the Covid-19 pandemic. Including the subsequent two years in our sample,[33] we find that the model continues to match the actual data well (left-panel in Figure 8). Looking at the contributions of shocks, our model suggests that the initial sharp downturn in 2020, as well as the subsequent jump in the year-over-year growth rate of aggregate productivity, are primarily attributable to a common shock (right-panel in Figure 8). This result is intuitive given the nature of the underlying pandemic shock, which entailed broad-based restrictions on social and economic activity.

[33]Given the unusual nature and size of the Covid-19 shock, we refrain from using the data in the period contaminated by the pandemic in our baseline analysis above, and the results in this section should be seen as illustrative.

Figure 8. Contributions to the growth puzzle: sectors vs. sectoral shocks (dashed)



Summary. Our production network framework yields several novel findings about the UK productivity slowdown. First, the majority of variation in aggregate productivity growth is driven by industry-specific shocks. Second, the common technology shock has been more important around the crisis years, and the Covid-19 pandemic, and its contributions have been generally more positive post-crisis than pre-crisis (excluding the Covid-19 period). Third, both in an accounting sense and as a source of industry-specific shocks, manufacturing has been by far the largest negative contributor to the UK's post-2010 slowdown in productivity growth. Finally, several sectors—most notably, administrative and support services activities, as well as a number of public services—have experienced notably more positive shocks post-crisis than their accounting contributions (reflecting possibly all shocks) would suggest.

6 Conclusion

In this paper, we have studied the relationship between industry size and centrality in a production network. In the UK, larger industries tend to be more central as input suppliers, and the two characteristics move in the same direction over the business cycle. We trace this positive co-movement to the elasticity of substitution across intermediate inputs, which we estimate to be around 0.35. At this value, technology shocks alone generate a counterfactually negative size-centrality relationship; demand-side shocks are needed to match the data. Our production network framework also allows us to go beyond standard accounting decompositions of the UK's post-2010 productivity slowdown by identifying the underlying shocks and tracing their propagation through the network. We find that idiosyncratic shocks to manufacturing more than account for the aggregate slowdown, while common technology shocks have partially offset it.

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Appendices

A. Data and Stylised Facts

A.1 Data

All data used in this paper are provided by the Office for National Statistics (ONS) and are publicly available.

Supply and Use Tables

The supply and use tables are published annually and available to download via this link. The vintage used in this paper was released on 29 October 2021.

Gross Value-Added (GVA) data

The data on industries' value-added can be obtained from the ONS's GDP(O) low-level aggregates dataset, available at this link. The vintage we use was released on 31 March 2022. The quarterly CVM GVA data are available from 1990Q1 to 2021Q4.

Jobs data

The data on jobs per industry can be downloaded via this link. The vintage we use was released on 15 March 2022. The quarterly jobs data are available from 1978Q2 to 2021Q4.

Deflators data

The data on industries' price deflators can be obtained via this link. The vintage we use was released on 15 March 2022.

Therefore, the full dataset used to obtain our baseline results and to filter out the shocks has $T = 91$ (1997Q2–2019Q4) and $N = 79$. The burn-in period is set to equal 7 quarters.

Seasonal adjustment

To seasonally adjust the data, we use the Census X-13 method in EViews.

A.2 Stylised Facts

This section presents selected details of stylised facts on UK production networks.

A.2.1 Weighted Outdegrees

Following equation (2) in the main text, the first-order (weighted) outdegree of industry j is defined as

$$D_{jt}^{1,\text{out}} := \sum_{i=1}^{N} \omega_{ijt}. \tag{A.1}$$

The second-order (weighted) outdegree of industry j is then defined as

$$D_{jt}^{2,\text{out}} := \sum_{i=1}^{N} \omega_{ijt} D_{it}^{1,\text{out}}. \tag{A.2}$$

While the first-order outdegree measures the importance of industry j as a *direct* input supplier to other industries, the second-order outdegree weights the input shares by the first-order outdegree of the *consuming* industry and is thus the first step towards measuring *indirect* linkages.

Figure A.1 (left panel) shows the empirical density function of first-order outdegrees. The distributions of outdegrees are skewed, with relatively heavy right tails (the empirical densities of second-order outdegrees, not shown here, are similarly fat-tailed). Relatedly, the right panel in Figure A.1 shows the empirical counter-cumulative distribution function (CCDF) of first-order outdegrees.[1] The horizontal axis is the first-order outdegree for each industry (shown on a log scale), and the vertical axis (also shown on a log scale) gives the probability that an industry has an outdegree larger than or equal any correspond x-axis value. The linearity of the rightmost part of the distribution, in which industries with the largest first-order outdegrees lie, suggests that it is well-approximated by a power law distribution.[2] In other words, a small number of input suppliers are responsible for supplying the bulk of intermediate inputs.

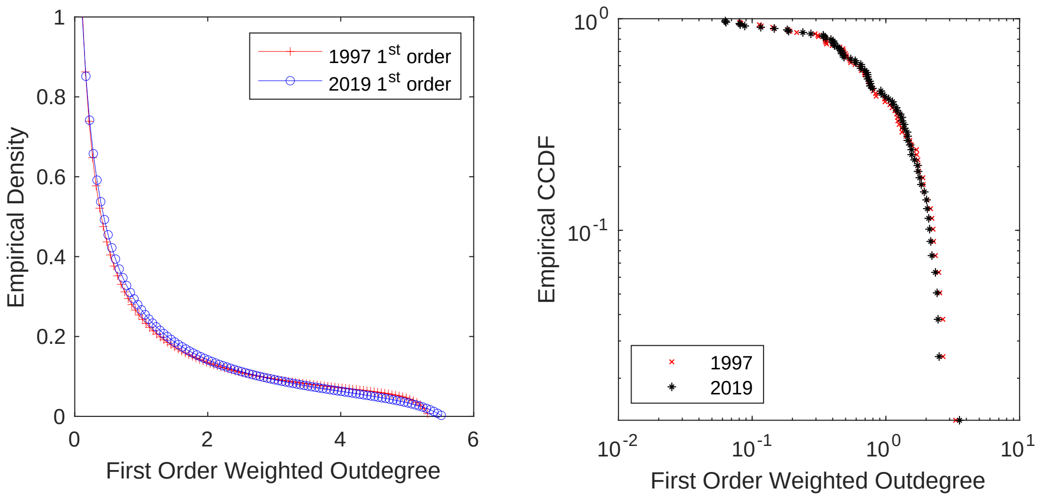
A.2.2 Bonacich Eigenvector Centrality

First-order and second-order outdegrees in equations (A.1) and (A.2) can be written in matrix form as $D_t^{1,\text{out}} = \mathbf{W}_t^\top \mathbf{1}$ and $D_t^{2,\text{out}} = (\mathbf{W}_t^\top)^2 \mathbf{1}$, respectively, where $\mathbf{1}$ denotes an $N \times 1$ vector of ones. We could analogously define higher-order outdegrees (e.g., third- and fourth-order outdegrees). Bonacich (1987) introduced a centrality measure that is closely related to the weighted outdegrees of all orders. More specifically, given parameters

[1] The overall skewness in the distribution of first-order outdegrees may reflect both skewness along the extensive margin (number of supplying linkages a producer has) and the intensive margin (how important the producer is as a supplier to another). Empirically, the vast majority of the skew in the distribution of first-order outdegrees is driven by the intensive margin, i.e., relatively few suppliers are responsible for supplying the bulk of material inputs.

[2] Carvalho (2014) obtains similar findings for the US.

Figure A.1. Empirical density and counter-cumulative distribution of first-order outdegrees in the UK



β_1 and β_2, the *Bonacich eigenvector centrality* (B_{jt}) of industry j is defined as:

$$B_{jt}(\beta_1, \beta_2) = \sum_{i=1}^{N} (\beta_1 + \beta_2 B_{it})\, \omega_{ijt}. \tag{A.3}$$

The parameter β_1 is a normalisation parameter and only affects the length of the vector of Bonacich centralities, whereas the parameter β_2 reflects the degree to which an industry's centrality is a function of the centrality of those industries to which it is connected. In matrix form, equation (A.3) is given by:

$$B_t = \beta_1 \mathbf{W}_t^\top \mathbf{1} + \beta_2 \mathbf{W}_t^\top B_t. \tag{A.4}$$

Solving for the vector of centralities B_t, we have that:

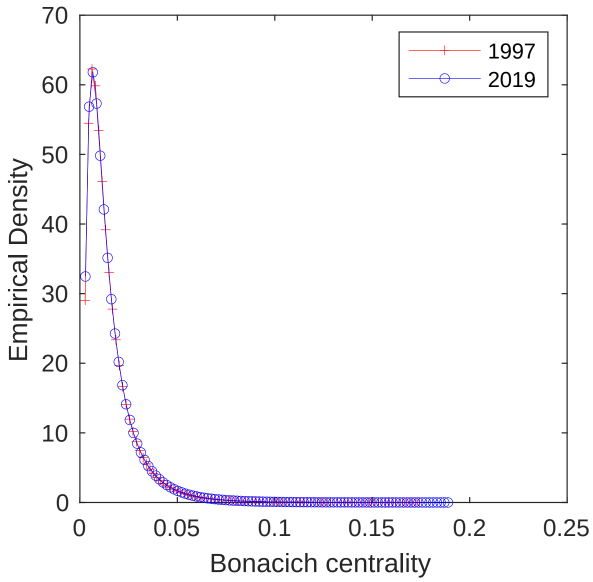
$$B_t = \beta_1 \left[\mathbf{I} - \beta_2 \mathbf{W}_t^\top\right]^{-1} \mathbf{W}_t^\top \mathbf{1}. \tag{A.5}$$

Using the power series expansion of the inverted matrix in equation (A.5), we have that the vector of Bonacich centralities is given by:

$$B_t = \left[\beta_1 \mathbf{W}_t^\top + \beta_1 \beta_2 (\mathbf{W}_t^\top)^2 + \dots\right] \mathbf{1}. \tag{A.6}$$

Therefore, the Bonacich eigenvector centrality is an infinite-order centrality measure in the sense that it captures both direct linkages (via first-order outdegrees) and indirect ones (via second-order outdegrees, third-order outdegrees, and so on). In other words, in our context, the Bonacich eigenvector centrality measure assigns to each industry a centrality score that is the sum of some baseline centrality level (common across all industries), and the centrality score of each of its downstream customers.[3] Figure A.2 shows that the distribution of Bonacich eigenvector centralities is also fat-tailed, with the right tail of its CCDF (not shown) close to linear on a log-log scale, and thereby well-approximated by a power law. Note also that the right tail of the empirical density of Bonacich centralities is more prominent than that shown in Figure A.1 for weighted outdegrees. Intuitively, once we take higher-order connectedness into account, that tends to further emphasise the role of central suppliers (in part because they also tend to supply inputs to more central industries).

Figure A.2. Empirical density of Bonacich centralities in the UK



We can visualise the heterogeneity in industries' importance as input suppliers by graphically representing the UK input-output network. In Figure A.3, each node corresponds to an industry and its size is proportional to the industry's Bonacich eigenvector centrality. The flows of inputs are represented by lines connecting the nodes, with the direction of the flow indicated by an arrow. Financial services (industry 64) are the most central input supplier in each year for which UK supply-and-use tables are available.[4] Several manufacturing industries are also very central, most notably manufacture of chemicals and chemical products (industry 20) and manufacture of basic metals (industry 24). Other very central industries include construction (industries 41-43), electricity, gas, steam and air conditioning supply (industry 35), computer programming, consultancy and related activities (industry 62), and employment activities (industry 78).[5] The majority of

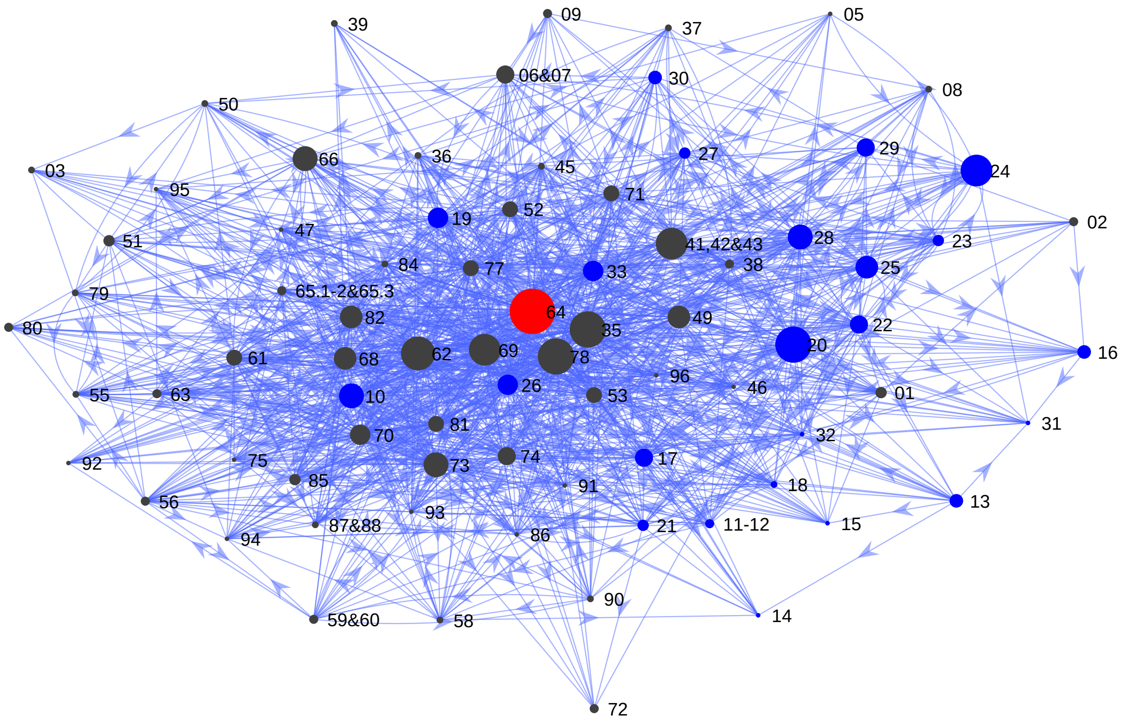
[3]Note that we set $\beta_1 = 0.5/N$ and $\beta_2 = 0.5$, where 0.5 is the share of intermediate inputs in production.

[4]These financial service activities exclude insurance and pension funding. The UK's supply-and-use tables are available over 1997-2019.

[5]Employment activities (industry 78) include activities of employment placement agencies and other human resources provision.

industries are, however, much less important as input suppliers to other industries. This corroborates the earlier claim that most industries supply highly-specialised inputs (to few other industries) whilst a few general-purpose input suppliers provide inputs to many other industries.

Figure A.3. A graphical representation of the UK input-output network in 2019



Notes: Finance (64) is the red node, manufacturing industries (10-33) are blue nodes.

A.2.3 Clustering in the Production Network

Beyond a visual inspection of Figure A.3, we can also analyse more formally the extent to which industries cluster in the input-output network. The clustering coefficient is a standard network topology measure that captures the tendency to which nodes in a graph (in our case, industries in the input-output network) tend to cluster together. We define the average clustering coefficient as:

$$AverageClustering_t = \frac{1}{N}\sum_{i=1}^{N}\frac{\frac{1}{2}\left[\left(\mathbf{W}_t+\mathbf{W}_t^T\right)\left(\mathbf{A}_t+\mathbf{A}_t^T\right)^2\right]_{ii}}{s_{i,t}^{tot}\left(d_{i,t}^{tot}-1\right)-2s_{i,t}^{\leftrightarrow}}, \tag{A.7}$$

where $\mathbf{A}_t$ is the *adjacency input-output matrix* whose entries a_{ijt} are a binary variable equal to one if $\omega_{ijt} > \tau$ and zero otherwise, for some chosen threshold τ,[6] $s_{i,t}^{tot} = (\mathbf{W}_t^T + \mathbf{W}_t)_i\mathbf{1}$, $d_{i,t}^{tot} = (\mathbf{A}_t^T + \mathbf{A}_t)_i\mathbf{1}$, and $s_{i,t}^{\leftrightarrow} = (\mathbf{W}_t\mathbf{A}_t + \mathbf{A}_t\mathbf{W}_t)_{ii}/2$, and where the subscript i (ii) denotes

[6]Below, we set $\tau = 1\%$, the same parametrisation as in Acemoglu et al. (2012).

the ith row (iith entry) in the corresponding matrix, and $\mathbf{1}$ denotes an N-dimensional vector of ones.[7] In a 'star economy', with one producer sourcing inputs from many other producers who are not themselves connected, the clustering coefficients would all be 0. On the other hand, if each industry sourced a fraction equal to $1/(N-1)$ of its inputs from *all* others (excluding itself), then all clustering coefficients would equal 1. Using our data, we find that the average clustering coefficient is very stable over time, with a mean of 0.84, suggesting that the UK's input-output network has a very high level of clustering.[8] More formally, this confirms the visual observation from Figure A.3 that most 'triangles' (sets of three interconnected industries) include industries with higher centrality.

A.2.4 Stability of Input–Output Linkages

We analyse how the time-variation in the input-output network relates to the correlation between different industries' output growth. Intuitively, one may expect that the higher the importance of industry j as a supplier of inputs to industry i (so the higher is ω_{ijt}) and/or the more stable this linkage is (so the lower is $\sigma(\omega_{ijt})$), the stronger the co-movement between the output of industries i and j. First, we estimate the following regression:

$$\rho(g_{ij}) = \alpha + \beta\overline{\omega}_{ij} + \varepsilon_{ij}, \quad i \neq j, \tag{A.8}$$

where $\rho(g_{ij})$ denotes the correlation between the growth rates of gross output in industries i and j and $\overline{\omega}_{ij}$ denotes the average share of inputs from industry j in industry i's total intermediate consumption. We find $\hat{\beta} = 1.38$, statistically significant at the 1% level. The pairwise correlations of gross output growth across industries tend to be higher the higher is the average share of inputs they source from each other: a 1pp increase in the average input share tends to be associated with close to a 0.014 increase in the correlation coefficient of gross output growth rates.[9] Next, we consider the following regression:

$$\rho(g_{ij}) = \alpha + \beta\overline{\omega}_{ij} + \gamma\hat{\sigma}(\omega_{ij}) + \varepsilon_{ij}, \quad i \neq j, \tag{A.9}$$

where $\hat{\sigma}(\omega_{ij})$ denotes the sample variance of ω_{ijt}. We now find that $\hat{\beta}$ increases to 1.66 (and is still statistically significant at the 1% level), and $\hat{\gamma} = -1.82$ (and is significant at the 5% level). Intuitively, in equation (A.8), the coefficient on the average share of inputs ($\bar{\omega}_{ij}$) was likely biased downwards, which is due to higher shares also being more volatile

[7]This definition follows Clemente and Grassi (2018). Note that self-loops are not considered, i.e. the main diagonals in $\mathbf{W}$ and $\mathbf{A}$ are replaced with zeros.

[8]To put this into a perspective, Clemente and Grassi (2018) find that a network consisting of banks from the core 24 countries (as defined by the Bank for International Settlements (BIS)) has a clustering coefficient of around 0.9.

[9]This finding is in line with Carvalho (2014), though based on $\mathbf{W}_t$ rather than the network distance measure Carvalho (2014) uses, which is based on $\mathbf{A}_t$.

and higher volatility decreasing the correlation between output growth rates. In other words, once we have controlled for the average share of inputs industries source from each other, the pairwise correlations of gross output growth across industries tend to be lower for those pairs where the respective share of inputs is more volatile. Intuitively, insofar as the higher volatility of ω_{ijt} indicates greater substitutability across input suppliers, a high elasticity of substitution implies that even small changes in relative prices translate into large changes in input shares, increasing $\sigma(\omega_{ijt})$.

B. Derivations Related to the Simplified Model

B.1 Competitive Equilibrium

A *competitive equilibrium* of this economy consists of prices $(P_{1t}, ..., P_{Nt}, W_t)$, labour $(L_{1t}, ..., L_{Nt})$, consumption $(C_{1t}, ..., C_{Nt})$, and intermediate inputs $(M_{ijt})_{i,j\in\{1,...,N\}}$ such that the representative household maximises her utility (subject to the budget constraint), the representative firms in each industry maximise profits, markets clear and the resource constraints are satisfied, that is

$$C_{it} + \sum_{j=1}^{N} M_{ijt} = Q_{it}, \qquad \forall i = 1, ..., N, \tag{B.1}$$

$$\sum_{i=1}^{N} P_{it}C_{it} = W_t L_t + \sum_{i=1}^{N} \Pi_{it}, \tag{B.2}$$

$$\sum_{i=1}^{N} L_{it} = L_t. \tag{B.3}$$

The household's problem is

$$\max_{\{C_{it}\}_{i\in(1,...,N)},\, L_t} \log \prod_{i=1}^{N} C_{it}^{\gamma_{it}} - \frac{\varepsilon_{LS}}{\varepsilon_{LS}+1} L_t^{\frac{\varepsilon_{LS}+1}{\varepsilon_{LS}}} \qquad \text{s.t.} \qquad \sum_{i=1}^{N} P_{it}C_{it} = W_t L_t + \sum_{i=1}^{N} \Pi_{it}, \tag{B.4}$$

Letting ν_t denote the Lagrange multiplier on the budget constraint, the first-order condition (FOC) with respect to C_{it} is given by

$$\frac{\gamma_{it}}{C_{it}} = \nu_t P_{it}. \tag{B.5}$$

The FOC w.r.t. L_t is given by

$$L_t^{\frac{1}{\varepsilon_{LS}}} = \nu_t W_t. \tag{B.6}$$

Multiplying the FOC w.r.t. C_{it} by C_{it}, summing over i and using the budget constraint together with $\sum_{i=1}^{N} \gamma_{it} = 1$, we obtain $\nu_t = 1/\big(W_t L_t + \sum_{i=1}^{N} \Pi_{it}\big)$, so the optimal demand for good i is

$$C_{it} = \gamma_{it} \frac{W_t L_t + \sum_{k=1}^{N} \Pi_{kt}}{P_{it}}. \tag{B.7}$$

Substituting $\nu_t = 1/\big(W_t L_t + \sum_{i=1}^{N} \Pi_{it}\big)$ into the FOC w.r.t. L_t yields the household's labour supply equation:

$$L_t^{\frac{1}{\varepsilon_{LS}}} = \frac{W_t}{W_t L_t + \sum_{i=1}^N \Pi_{it}}. \tag{B.8}$$

When firms earn positive profits ($\eta_i < 1$ for some i), the labour share of household income, $W_t L_t / (W_t L_t + \sum_{i=1}^N \Pi_{it})$, is strictly less than one, so the equilibrium labour supply L_t is strictly less than one.[10] The ideal price index associated with the consumption aggregator $C_t \coloneqq \prod_{i=1}^N C_{it}^{\gamma_{it}}$ is $\mathrm{P}_t = \prod_{i=1}^N (P_{it}/\gamma_{it})^{\gamma_{it}}$.

To produce a given amount Q_{it}, the representative firm in each industry i faces the following cost-minimisation problem:

$$\min_{L_{it},\, \{M_{ijt}\}_{j\in\{1,\ldots,N\}}} W_t L_{it} + \sum_{j=1}^N P_{jt} M_{ijt} - \Lambda_{it} \left[\left(A_{it} L_{it}^{1-\mu_i} M_{it}^{\mu_i} \right)^{\eta_i} - Q_{it} \right].$$

The FOC with respect to L_{it} is

$$W_t - \Lambda_{it} \left(\eta_i (1-\mu_i) L_{it}^{\eta_i(1-\mu_i)-1} A_{it}^{\eta_i} M_{it}^{\eta_i \mu_i} \right) = 0. \tag{B.9}$$

Multiplying both sides by L_{it}, we have that:

$$W_t L_{it} = \Lambda_{it} \eta_i (1-\mu_i) Q_{it}, \qquad \text{i.e.} \qquad L_{it} = \frac{\Lambda_{it} \eta_i (1-\mu_i) Q_{it}}{W_t}. \tag{B.10}$$

where Λ_{it} is the Lagrange multiplier, and the FOC with respect to M_{ijt} is

$$P_{jt} - \Lambda_{it} \left(A_{it}^{\eta_i} L_{it}^{\eta_i(1-\mu_i)} \frac{\eta_i \mu_i \varepsilon_M}{\varepsilon_M - 1} M_{it}^{\frac{\eta_i \mu_i \varepsilon_M - \varepsilon_M + 1}{\varepsilon_M}} \left(\Gamma_{ij}^M \right)^{\frac{1}{\varepsilon_M}} \frac{\varepsilon_M - 1}{\varepsilon_M} M_{ijt}^{\frac{-1}{\varepsilon_M}} \right) = 0, \tag{B.11}$$

which simplifies to

$$P_{jt} - \Lambda_{it} \left(A_{it}^{\eta_i} L_{it}^{\eta_i(1-\mu_i)} \eta_i \mu_i M_{it}^{\frac{\eta_i \mu_i \varepsilon_M - \varepsilon_M + 1}{\varepsilon_M}} \left(\Gamma_{ij}^M \right)^{\frac{1}{\varepsilon_M}} M_{ijt}^{\frac{-1}{\varepsilon_M}} \right) = 0. \tag{B.12}$$

Multiplying both sides by M_{ijt} and rearranging,

$$M_{ijt} = \left(\frac{\Lambda_{it} \eta_i \mu_i Q_{it} M_{it}^{\frac{1-\varepsilon_M}{\varepsilon_M}} \left(\Gamma_{ij}^M \right)^{\frac{1}{\varepsilon_M}}}{P_{jt}} \right)^{\varepsilon_M}. \tag{B.13}$$

Let P_{it}^M denote the ideal price index associated with the CES intermediate input

[10] In the special case of constant returns to scale ($\eta_i = 1$ for all i), profits vanish and the FOC collapses to $L_t^{1/\varepsilon_{LS}} = 1/L_t$, which yields $L_t = 1$.

bundle M_{it}:

$$P_{it}^M = \left[\sum_{j=1}^{N} \Gamma_{ij}^M P_{jt}^{1-\varepsilon_M}\right]^{\frac{1}{1-\varepsilon_M}}. \tag{B.14}$$

Using the FOC with respect to M_{ijt}, it follows that

$$P_{it}^M = \Lambda_{it}\eta_i\mu_i Q_{it} M_{it}^{-1}, \tag{B.15}$$

or, equivalently,

$$M_{it} = \frac{\Lambda_{it}\eta_i\mu_i Q_{it}}{P_{it}^M}. \tag{B.16}$$

Plugging the cost-minimising expressions for L_{it} and M_{it} back into the production function yields

$$Q_{it} = A_{it}^{\eta_i}\left(\frac{\Lambda_{it}\eta_i(1-\mu_i)Q_{it}}{W_t}\right)^{\eta_i(1-\mu_i)}\left(\frac{\Lambda_{it}\eta_i\mu_i Q_{it}}{P_{it}^M}\right)^{\eta_i\mu_i}. \tag{B.17}$$

Solving for Λ_{it},

$$\Lambda_{it} = \zeta_i \frac{Q_{it}^{\frac{1-\eta_i}{\eta_i}}}{A_{it}} W_t^{1-\mu_i}(P_{it}^M)^{\mu_i}, \tag{B.18}$$

where $\zeta_i = [\eta_i\mu_i]^{-\mu_i}\,[\eta_i(1-\mu_i)]^{-(1-\mu_i)}$. The cost-minimising solutions for L_{it}, M_{it}, and M_{ijt} are then

$$L_{it} = \frac{\zeta_i\eta_i(1-\mu_i)Q_{it}^{\frac{1}{\eta_i}} W_t^{-\mu_i}(P_{it}^M)^{\mu_i}}{A_{it}}, \tag{B.19}$$

$$M_{it} = \frac{\zeta_i\eta_i\mu_i Q_{it}^{\frac{1}{\eta_i}} W_t^{1-\mu_i}(P_{it}^M)^{\mu_i-1}}{A_{it}}, \tag{B.20}$$

$$M_{ijt} = \frac{\zeta_i\eta_i\mu_i Q_{it}^{\frac{1}{\eta_i}} W_t^{1-\mu_i}(P_{it}^M)^{\mu_i-1+\varepsilon_M}\Gamma_{ij}^M}{A_{it}P_{jt}^{\varepsilon_M}}. \tag{B.21}$$

Therefore, the total cost at the optimum is

$$\begin{aligned} C(Q_{it}) &= W_t L_{it} + P_{it}^M M_{it} \\ &= Q_{it}^{\frac{1}{\eta_i}}\left(\frac{\zeta_i\eta_i(1-\mu_i)}{A_{it}} W_t^{1-\mu_i}(P_{it}^M)^{\mu_i} + \frac{\zeta_i\eta_i\mu_i W_t^{1-\mu_i}(P_{it}^M)^{\mu_i-1+\varepsilon_M}}{A_{it}}\sum_{j=1}^{N}\Gamma_{ij}^M P_{jt}^{1-\varepsilon_M}\right). \end{aligned} \tag{B.22}$$

Recalling $\zeta_i = [\eta_i\mu_i]^{-\mu_i}\,[\eta_i(1-\mu_i)]^{-(1-\mu_i)}$, the marginal cost is

$$C'(Q_{it}) = \frac{Q_{it}^{\frac{1-\eta_i}{\eta_i}}}{A_{it}} \left(\frac{W_t}{\eta_i(1-\mu_i)}\right)^{1-\mu_i} \left(\frac{P_{it}^M}{\eta_i\mu_i}\right)^{\mu_i}. \tag{B.23}$$

Since markets are perfectly competitive, $P_{it} = C'(Q_{it})$, i.e.,

$$P_{it} = \frac{Q_{it}^{\frac{1-\eta_i}{\eta_i}}}{A_{it}} \left(\frac{W_t}{\eta_i(1-\mu_i)}\right)^{1-\mu_i} \left(\frac{P_{it}^M}{\eta_i\mu_i}\right)^{\mu_i}. \tag{B.24}$$

At the cost-minimising allocation, total production cost satisfies $W_tL_{it}+P_{it}^M M_{it} = \Lambda_{it}\eta_i(1-\mu_i)Q_{it}+\Lambda_{it}\eta_i\mu_iQ_{it} = \eta_i\Lambda_{it}Q_{it}$. Combined with $P_{it} = \Lambda_{it}$ from perfect competition, the profit of industry i is therefore

$$\Pi_{it} \coloneqq P_{it}Q_{it} - W_tL_{it} - P_{it}^M M_{it} = (1-\eta_i)P_{it}Q_{it}. \tag{B.25}$$

Profits in industry i are thus a fraction $(1-\eta_i)$ of nominal gross output: positive whenever returns to scale are decreasing, and zero in the constant-returns-to-scale limit. Aggregating, $\sum_{i=1}^N \Pi_{it} = \sum_{i=1}^N (1-\eta_i)P_{it}Q_{it}$, so the household budget constraint can be written equivalently as $\sum_{i=1}^N P_{it}C_{it} = W_tL_t + \sum_{i=1}^N (1-\eta_i)P_{it}Q_{it}$.

Plugging the solutions for C_{it} (from the household's problem) and M_{jit} (from the firms' problem) into the market-clearing condition yields

$$Q_{it} = \frac{\gamma_{it}\big[W_tL_t + \sum_{k=1}^N (1-\eta_k)P_{kt}Q_{kt}\big]}{P_{it}} + \sum_{j=1}^N \frac{\zeta_j\eta_j\mu_jQ_{jt}^{\frac{1}{\eta_j}}W_t^{1-\mu_j}(P_{jt}^M)^{\mu_j-1+\varepsilon_M}\Gamma_{ji}^M}{A_{jt}P_{it}^{\varepsilon_M}}. \tag{B.26}$$

The equilibrium of this economy can therefore be expressed as a system of $3N+1$ equations in $3N+1$ unknowns, $\{(P_{it})_{i=1}^N, (Q_{it})_{i=1}^N, (L_{it})_{i=1}^N, W_t\}$, supplemented by $L_t = \sum_{i=1}^N L_{it}$:

$$P_{it} = \frac{Q_{it}^{\frac{1-\eta_i}{\eta_i}}}{A_{it}} \left(\frac{W_t}{\eta_i(1-\mu_i)}\right)^{1-\mu_i} \left(\frac{P_{it}^M}{\eta_i\mu_i}\right)^{\mu_i}, \tag{B.27}$$

$$Q_{it} = \frac{\gamma_{it}\big[W_tL_t + \sum_{k=1}^N (1-\eta_k)P_{kt}Q_{kt}\big]}{P_{it}} + \sum_{j=1}^N \frac{\zeta_j\eta_j\mu_jQ_{jt}^{\frac{1}{\eta_j}}W_t^{1-\mu_j}(P_{jt}^M)^{\mu_j-1+\varepsilon_M}\Gamma_{ji}^M}{A_{jt}P_{it}^{\varepsilon_M}}, \tag{B.28}$$

$$L_{it} = \frac{\zeta_i\eta_i(1-\mu_i)Q_{it}^{\frac{1}{\eta_i}}W_t^{-\mu_i}(P_{it}^M)^{\mu_i}}{A_{it}}, \tag{B.29}$$

$$L_t^{\frac{1}{\varepsilon_{LS}}} = \frac{W_t}{W_tL_t + \sum_{i=1}^N (1-\eta_i)P_{it}Q_{it}}. \tag{B.30}$$

The last equation is the household's labour supply condition, which replaces the result

$L_t = 1$ used in the constant-returns-to-scale case. To solve the model, we take the consumer price index, P_t, as the numeraire and normalise it to equal 1 without loss of generality, which yields an additional equation.

B.2 Equilibrium Intermediate Input Shares

By definition, the share of industry i's intermediate input expenditure attributable to goods sourced from industry j is

$$\omega_{ijt} := \frac{P_{jt} M_{ijt}}{P_{it}^M M_{it}}. \tag{B.31}$$

Using the optimal intermediate input demands derived in Appendix B.1, the equilibrium intermediate input shares are

$$\begin{aligned} \omega_{ijt} &= \frac{P_{jt}\left(\frac{\zeta_i \eta_i \mu_i Q_{it}^{\frac{1}{\eta_i}} W_t^{1-\mu_i} (P_{it}^M)^{\mu_i - 1 + \varepsilon_M} \Gamma_{ij}^M}{A_{it} P_{jt}^{\varepsilon_M}}\right)}{P_{it}^M \left(\frac{\zeta_i \eta_i \mu_i Q_{it}^{\frac{1}{\eta_i}} W_t^{1-\mu_i} (P_{it}^M)^{\mu_i - 1}}{A_{it}}\right)} \\ &= \Gamma_{ij}^M \left(\frac{P_{jt}}{P_{it}^M}\right)^{1-\varepsilon_M}. \end{aligned} \tag{B.32}$$

B.3 First-Order and Second-Order Weighted Outdegrees

The first-order weighted outdegree of industry j is

$$D_{jt}^{1,\text{out}} := \sum_{i=1}^{N} \omega_{ijt} = \sum_{i=1}^{N} \Gamma_{ij}^M \left(\frac{P_{jt}}{P_{it}^M}\right)^{1-\varepsilon_M}. \tag{B.33}$$

The second-order weighted outdegree of industry j is

$$D_{jt}^{2,\text{out}} := \sum_{i=1}^{N} \omega_{ijt} D_{it}^{1,\text{out}}. \tag{B.34}$$

We now characterise the responses of first-order and second-order weighted outdegrees to technology and demand shocks. By definition,

$$\frac{\partial D_{jt}^{2,\text{out}}}{\partial P_{jt}} = \sum_{i=1}^{N} \left[\frac{\partial \omega_{ijt}}{\partial P_{jt}} D_{it}^{1,\text{out}} + \omega_{ijt} \frac{\partial D_{it}^{1,\text{out}}}{\partial P_{jt}}\right].$$

Since P_{jt} has a weight of at most one in P_{it}^M, the first partial derivative $(\partial \omega_{ijt} / \partial P_{jt})$ is non-negative, and it will generally be positive as long as industry i sources inputs from industries other than industry j as well. For the second partial derivative, note that

$$\frac{\partial D_{it}^{1,\text{out}}}{\partial P_{jt}} = \sum_{k=1}^{N} \left[\Gamma_{ki}^{M} P_{it}^{1-\varepsilon_M} (\varepsilon_M - 1)(P_{kt}^{M})^{\varepsilon_M - 2} \Gamma_{kj}^{M} P_{jt}^{-\varepsilon_M} (P_{kt}^{M})^{\varepsilon_M}\right],$$

which is clearly negative if $\varepsilon_M < 1$. The above equation can be rewritten as

$$\frac{\partial D_{it}^{1,\text{out}}}{\partial P_{jt}} = \sum_{k=1}^{N} \left[\omega_{kit}(\varepsilon_M - 1)\omega_{kjt} P_{jt}^{-1}\right].$$

If $i = j$, then

$$\frac{\partial D_{jt}^{1,\text{out}}}{\partial P_{jt}} = \sum_{i=1}^{N} (1 - \varepsilon_M)\omega_{ijt}(1 - \omega_{ijt}) P_{jt}^{-1}.$$

Therefore, the partial derivative of industry j's second-order weighted outdegree with respect to its price is

$$\begin{aligned}
\frac{\partial D_{jt}^{2,\text{out}}}{\partial P_{jt}} &= \sum_{i=1}^{N} \frac{\partial \omega_{ijt}}{\partial P_{jt}} D_{it}^{1,\text{out}} + \sum_{i=1}^{N} \omega_{ijt} \frac{\partial D_{it}^{1,\text{out}}}{\partial P_{jt}} \\
&= \sum_{i=1}^{N} (1 - \varepsilon_M)\omega_{ijt}(1 - \omega_{ijt}) P_{jt}^{-1} \sum_{k=1}^{N} \omega_{kit} + \sum_{i=1, i \neq j}^{N} \omega_{ijt} \sum_{k=1}^{N} \left[\omega_{kit}(\varepsilon_M - 1)\omega_{kjt} P_{jt}^{-1}\right] \\
&\quad + \omega_{jjt} \sum_{i=1}^{N} (1 - \varepsilon_M)\omega_{ijt}(1 - \omega_{ijt}) P_{jt}^{-1} \\
&= (1 - \varepsilon_M) P_{jt}^{-1} \sum_{i=1, i \neq j}^{N} \omega_{ijt} \sum_{k=1}^{N} \omega_{kit} \left(1 - \omega_{ijt} - \omega_{kjt}\right) \\
&\quad + (1 - \varepsilon_M) P_{jt}^{-1} \omega_{jjt} \sum_{k=1}^{N} \omega_{kjt} \left(2 - \omega_{jjt} - \omega_{kjt}\right),
\end{aligned}$$

which can be rewritten as

$$\frac{\partial D_{jt}^{2,\text{out}}}{\partial P_{jt}} = (1 - \varepsilon_M) P_{jt}^{-1} \left[\sum_{i=1}^{N} \omega_{ijt} \sum_{k=1}^{N} \omega_{kit} \left(1 - \omega_{ijt} - \omega_{kjt}\right) + \omega_{jjt} \sum_{k=1}^{N} \omega_{kjt}\right].$$

This derivative is non-negative if

$$\begin{aligned}
\sum_{i=1}^{N} \omega_{ijt} \sum_{k=1}^{N} \omega_{kit} \left(1 - \omega_{ijt} - \omega_{kjt}\right) + \sum_{i=1}^{N} \omega_{ijt}\omega_{jjt} \geq 0, \qquad \text{i.e.} \\
\sum_{i=1}^{N} \omega_{ijt}\omega_{jjt} \geq \sum_{i=1}^{N} \omega_{ijt} \sum_{k=1}^{N} \omega_{kit} \left(\omega_{ijt} + \omega_{kjt} - 1\right).
\end{aligned}$$

The above condition can be written as

$$D_{jt}^{2,\text{out}} + \omega_{jjt} D_{jt}^{1,\text{out}} \geq \sum_{i=1}^{N} \omega_{ijt} \sum_{k=1}^{N} \omega_{kit} \left(\omega_{ijt} + \omega_{kjt}\right), \tag{B.35}$$

which is strictly satisfied if $\omega_{ijt} + \omega_{kjt} < 1$, which will typically be the case. Note that if $\omega_{ijt} + \omega_{kjt} = 1$, then the right-hand side equals $D_{jt}^{2,\text{out}}$.

Note that the maximum value of $\omega_{ijt} + \omega_{kjt}$ for any $(i, k) \in N$ pair is 2. For a given ω_{kjt}, the maximum value of ω_{kit} is $1 - \omega_{kjt}$. So the maximum value of the right-hand side in equation (B.35) is given by the right-hand side of the inequality below:

$$\sum_{i=1}^{N} \omega_{ijt} \omega_{jjt} > \sum_{i=1}^{N} \omega_{ijt} \sum_{k=1}^{N} (1 - \omega_{kjt}) \left(\omega_{ijt} + \omega_{kjt} - 1\right).$$

Suppose that $\omega_{jjt} = 0$, which makes the left-hand side of the above inequality as small as possible. For the inequality to still hold, we need

$$\sum_{i=1}^{N} \omega_{ijt} \sum_{k=1, k \neq j}^{N} (1 - \omega_{kjt}) \left(\omega_{ijt} - (1 - \omega_{kjt})\right) + \sum_{i=1}^{N} \omega_{ijt} (1 - \omega_{jjt}) \left(\omega_{ijt} - (1 - \omega_{jjt})\right) < 0,$$

or, equivalently,

$$\sum_{i=1}^{N} \omega_{ijt} \sum_{k=1, k \neq j}^{N} \left[(1 - \omega_{kjt}) \left(\omega_{ijt} - \omega_{ijt}(1 - \omega_{kjt})\right) - (1 - \omega_{ijt})\right] < 0. \tag{B.36}$$

Treating ω_{ijt} as given, the expression inside the second sum is maximised when

$$1 - \omega_{kjt} - \omega_{ijt} - \omega_{kjt} + 1 = 0, \qquad \text{i.e.} \qquad \omega_{kjt} = \frac{2 - \omega_{ijt}}{2}.$$

If $k = i$, then clearly $\omega_{kjt} = 2/3$. It can then be easily shown that the term inside the second sum in equation (B.36) is given by

$$(1 - \omega_{kjt}) \left(\omega_{ijt} - (1 - \omega_{kjt})\right) - (1 - \omega_{ijt}) = -\frac{3}{4} \omega_{ijt}^2 - \omega_{ijt} < 0 \quad \text{if} \quad \omega_{ijt} \neq 0.$$

Intuitively, since ω_{ijt} increases with P_{jt} for all i, then at least some of the remaining shares ω_{ikt} $(k \neq j)$ have to fall. This reduces other industries' $(k \neq j)$ outdegrees, but the sum of all outdegrees must always equal N, by definition. For industry j, the increases in ω_{ijt} more than offset the falls in $D_{it}^{1,\text{out}}$, so $D_{jt}^{2,\text{out}}$ is increasing in P_{jt}.

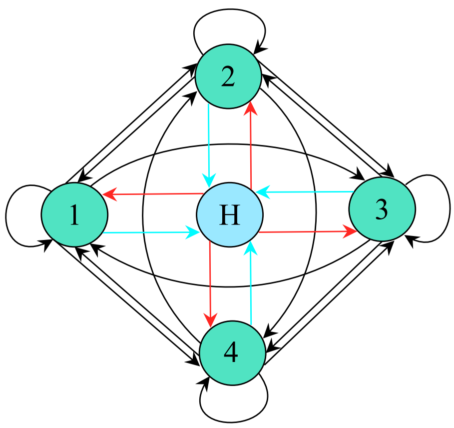
B.4 Size-Centrality Relationship in Two Simple Economies

Consider two simple economies consisting of four industries and a representative household. In both examples, we assume that (i) the intermediate-input share in gross output equals 60% for all industries (i.e. $\mu_i = \mu = 0.6$), (ii) there are slightly decreasing returns to scale ($\eta_i = 0.95$), implying that preference shocks affect relative prices, (iii) intermediate inputs are gross complements ($\varepsilon_M = 0.4$), and (iv) the Frisch elasticity of labour supply equals 2 (i.e. $\varepsilon_{LS} = 2$). The two crucial parameters for the purposes of this exercise are ε_M and η_i: $\varepsilon_M < 1$ implies that shocks to industry j that lower (raise) its price will lower (raise) its centrality as measured by its outdegree, and $\eta_i < 1$ implies that relative prices (and thus also outdegrees) need not be invariant to preference shocks.[11]

Example 1: Symmetric Economy

Consider a $N = 4$ economy consisting of identical industries and a representative household, i.e. it is perfectly symmetric (Figure B.1). In particular, assume that the steady-state intermediate input shares $\Gamma^M_{ij} = 1/4$ for all (i, j) pairs, and that the steady-state consumption shares $\xi_i = 1/4$ for all i. We further assume that all shocks follow an AR(1) process, and consider separately a 20% increase in A_{1t} at time $t = 1$, and a 20% increase in D_{1t} at time $t = 1$. The effect of these innovations will die out gradually as we set the AR(1) coefficient to 0.9.

Figure B.1. Graphical representation of the economy in Example 1



Notes: Input-output linkages are denoted with black arrows. The supply of labour from the household to producers is denoted with red arrows. The supply of consumption goods to the household is denoted with cyan arrows.

Figure B.2 shows the responses to the positive technology shock to industry 1.

[11] If $\eta_i = 1$, preference shocks will have no effect on relative prices. If $\varepsilon_M = 1$, industries' centralities will be independent of shocks.

This shock will make industry 1 relatively more productive than the other industries, driving its real output up and its price down. Its outdegree and Domar weight fall, and the opposite happens to the other industries, whose responses are identical given the symmetric nature of this economy. Intuitively, since P_{1t} falls by more than the other prices upon impact, the outdegree of industry 1 falls. Although the real output of all industries increases, nominal output of industry 1 increases by less due to the offsetting effect of its price falling by more, and hence its Domar weight falls upon impact. In other words, idiosyncratic technology shocks imply a negative (positive) relationship between output (Domar weights) and centrality for the industry in which the shock originates. For the other industries, the implied size-centrality relationship is positive.

Figure B.2. Responses to a positive technology shock in industry 1 (Example 1)

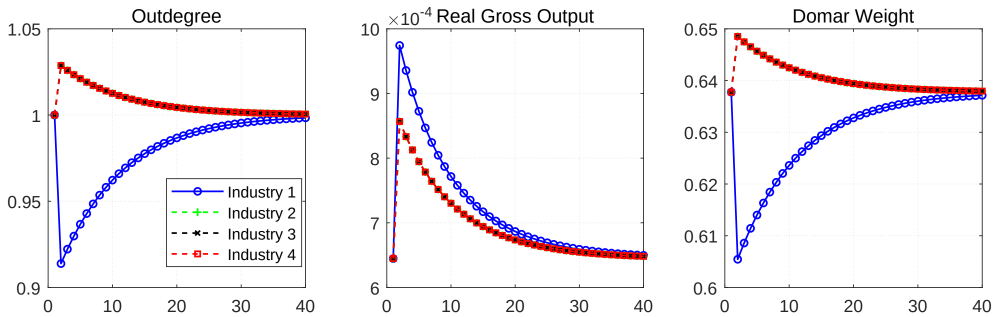
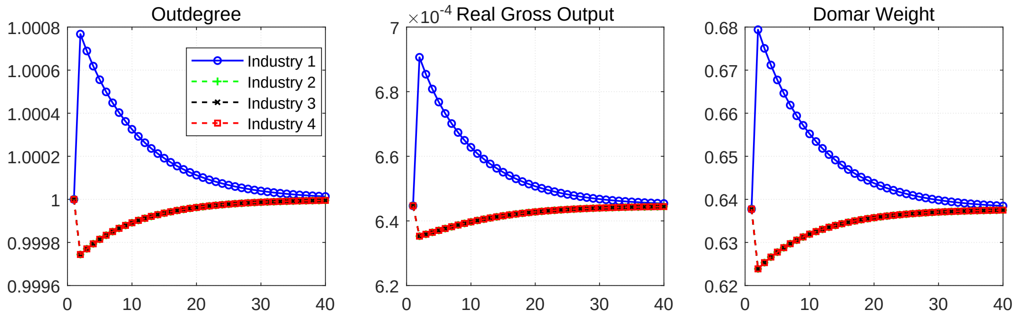


Figure B.3 shows the responses to the positive demand shock in industry 1. The exogenous increase in the household's 'taste' for the output of industry 1 will drive its real output and price up, so industry 1's outdegree, real output, and Domar weight all increase. The opposite happens to the other industries. Intuitively, since P_{1t} increases by more than the other prices, industry 1's outdegree increases. Therefore, idiosyncratic preference shocks imply a positive size-centrality relationship for all industries. Note, however, that although the demand shock in industry 1 increases its outdegree, the effect is very small. Intuitively, demand shocks leave relative prices unchanged in this economy *as long as* there are constant returns to scale.[12] Since we assume slightly decreasing returns to scale ($\eta_i = 0.95$), demand shocks do affect prices, but only slightly.

[12] If $\eta_i = 1$, we can solve for the relative prices, independently of the demand shocks.

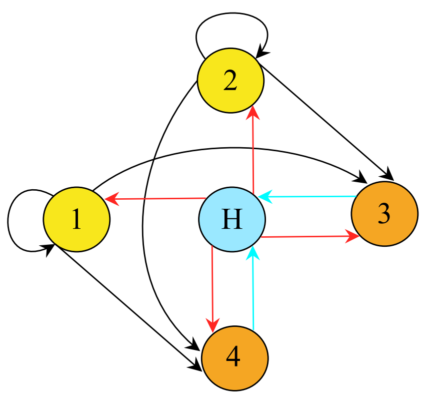
Figure B.3. Responses to a positive demand shock in industry 1 (Example 1)



Example 2: Separate Production of Intermediate and Final Goods

Consider now a different $N = 4$ economy consisting of industries 1 and 2, which produce intermediate goods *only* without sourcing from each other, and industries 3 and 4, which produce final goods *only* and source their intermediate inputs from industries 1 and 2 (Figure B.4). Suppose further that industry 2 is relatively more important as an input supplier than industry 1. We will parametrise this economy by setting the intermediate input shares $\Gamma^M_{11} = \Gamma^M_{22} = 1$, $\Gamma^M_{31} = \Gamma^M_{41} = 0.3$, $\Gamma^M_{32} = \Gamma^M_{42} = 0.7$, and $\Gamma^M_{ij} = 0$ for all other (i, j) pairs. Assume further that industry 3 is relatively less important as a final good producer than industry 4, i.e. $\xi_3 = 0.3$. We first consider a 20% increase in A_{1t} at time $t = 1$, and then separately a 20% increase in D_{3t} at time $t = 1$.

Figure B.4. Graphical representation of the economy in Example 2



Notes: Input-output linkages are denoted with black arrows. The supply of labour from the household to producers is denoted with red arrows. The supply of consumption goods to the household is denoted with cyan arrows.

Figure B.5 shows the responses to a positive technology shock to industry 1. This shock makes one of the two input-supplying industries relatively more productive than before, reducing its marginal cost and allowing it to set a lower price. Since industry 1's price falls by more than that of industry 2, industry 1's outdegree falls and that of industry 2 (equal to $4 - D_{1t}^{1,\text{out}}$) increases. Final-good-producing industries 3 and 4 have, by construction, zero outdegrees. Real output increases across all industries. The Domar weight of industry 1 falls and that of industry 2 increases. The Domar weights of the final good producers are unchanged since they only depend on household preferences:

$$\frac{P_{it}Q_{it}}{\sum_{k=1}^{N} P_{kt}C_{kt}} = \frac{P_{it}C_{it}}{\sum_{k=1}^{N} P_{kt}C_{kt}} = \gamma_{it}, \qquad i = 3, 4. \tag{B.37}$$

which is a consequence of the Cobb-Douglas nature of household preferences. Therefore, in this economy which features a completely separate production of intermediate and final goods, the implied size-centrality relationship due to technology shocks to intermediate producers is the same as in the symmetric economy in example 1.[13]

Figure B.5. Responses to a positive technology shock in industry 1 (Example 2)

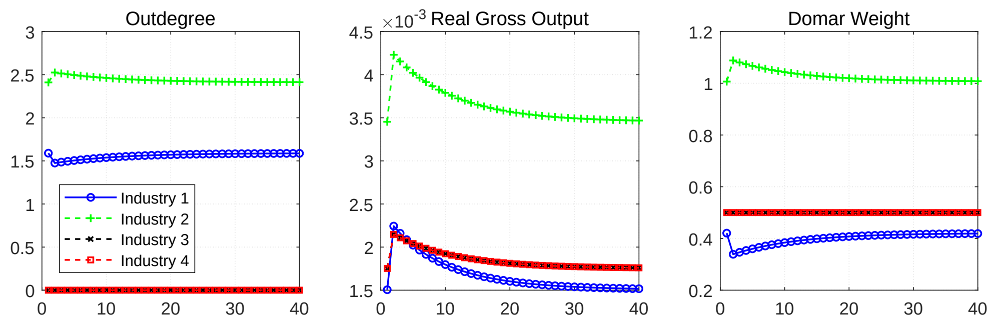
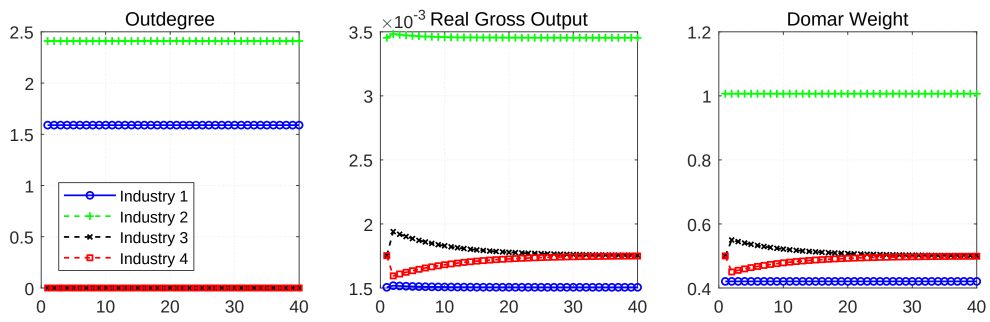


Figure B.6 shows the impulse responses to a positive demand shock in industry 3. This shock increases the household's taste for the output of industry 3. Note that the outdegrees of all industries are unchanged. Intuitively, the outdegrees of industries 1 and 2 are a function of P_{1t} and P_{2t} only (and some parameters), and the relative price P_{1t}/P_{2t} is constant as long as there are no technology shocks to industries 1 and 2. The figure also shows that real output of industry 3 increases, whilst that of industry 4 falls; this is expected given the 'shift' in household preferences. Real gross output of the input-supplying industries 1 and 2 increases slightly. Finally, whilst the Domar weight of industry 3 increases and that of industry 4 falls, the Domar weights of the input suppliers are unchanged. Note that the Domar weights of industries 1 and 2 are independent of all shocks in the model, and do not change because the relative price

[13] The final-good producers in this economy have, by construction, zero centrality. Since technology shocks to them do not affect the relative price of the intermediate-good producers, they imply no relationship between size and centrality for any industry.

P_{1t}/P_{2t} is independent of demand shocks.[14] Therefore, if the production of intermediate and final goods is completely separated, demand shocks imply no relationship between size and centrality (since centrality is independent of them).

Figure B.6. Responses to a positive demand shock in industry 3 (Example 2)



Overall, these two examples based on the simple model from Section 3 suggest that, in general, technology shocks tend to imply a negative relationship between real output and outdegrees, and a positive one between Domar weights and outdegrees.[15] Demand shocks, on the other hand, tend to imply a positive size-centrality relationship regardless of whether one takes output or Domar weight as the measure of size.

B.5 Two Generalisations of the Elasticity of Substitution Across Intermediate Inputs

Generalisation 1: Different ε_M Across Producers

We implicitly assumed above that ε_M is identical across all producers. Allowing ε_M to differ across producers would simply require replacing ε_M by the producer-specific ε_M^i in equation (3) in the main text.[16] This is because the only elasticities that appear in the cost-minimisation problem of each producer are those characterising its own production process. The effect of changing P_{jt} on P_{it}^M in equation (B.33) will depend on ε_M^i (and not ε_M^j), but since the weight of P_{jt} in P_{it}^M will typically be less than one, the change in P_{it}^M will tend to be less than proportional, at least in the range of empirically plausible values of ε_M^i. The intuition from above would thus apply: as long as $\varepsilon_M^i < 1$, a shock that

[14]If there are also constant returns to scale, the Domar weights of industries 1 and 2 sum to $\mu/(1-\mu)$.

[15]As Example 1 shows, technology shocks may imply a positive size-centrality relationship for the industries *other than* the industry in which the shock originated. However, in more realistic settings such as that in Section 4, the majority of variation in industries' size and centrality will tend to be driven by their *own* shocks, hence our emphasis on the implied relationship for the 'shock-originating' industry.

[16]Similarly, we assume throughout that ε_M is time-invariant. Relaxing this assumption would amount to replacing ε_M by $\varepsilon_{M,t}$ in equation (B.33), with the underlying logic unchanged.

causes industry j's own price P_{jt} to increase (decrease) will lead to an increase (decrease) in industry j's outdegree.

Generalisation 2: More vs. Less Substitutable Intermediate Inputs

Another assumption that we made above is that within the bundle of intermediate inputs (M_{it}), all inputs are equally substitutable (or complementary). One might instead argue that, for a given producer, there may exist multiple kinds of intermediate inputs. More specifically, suppose that each producer faces a set of (almost) essential intermediate inputs, $\mathcal{S}$, and a set consisting of the remaining intermediate inputs, $\mathcal{N}$. Intermediate inputs such as electricity, gas, and water could plausibly belong to the former set, and others such as water transport and air transport could belong to the latter set. Producers would then bundle together the two sets of inputs, each consisting of (different) intermediate inputs themselves. Assuming the bundling is consistent with CES aggregation, the bundle (M_{it}) would be characterised by three elasticities of substitution. First, substitutability within the two sets of inputs would be determined by potentially different elasticities $\varepsilon_{\mathcal{S}}$ and $\varepsilon_{\mathcal{N}}$, respectively. Second, the extent to which producers can substitute across the two sets of inputs would be determined by $\varepsilon_{\mathcal{SN}}$.

In Appendix B.6, we show that the first-order weighted outdegree of industry j belonging to set $\varkappa = \{\mathcal{S}, \mathcal{N}\}$ is given by:

$$D_{jt}^{1,\text{out}} = \sum_{i=1}^{N} \Gamma_{ij}^{M} \left(\frac{P_{jt}}{P_{it}^{\varkappa}}\right)^{1-\varepsilon_{\varkappa}} \left(\frac{P_{it}^{\varkappa}}{P_{it}^{M}}\right)^{1-\varepsilon_{\mathcal{SN}}}, \tag{B.38}$$

where $\varepsilon_{\varkappa}$ corresponds to $\varepsilon_{\mathcal{S}}$ ($\varepsilon_{\mathcal{N}}$) if $j \in \mathcal{S}$ ($j \in \mathcal{N}$), and similarly for $P_{it}^{\varkappa}$, which denotes the ideal price index associated with the bundle of (almost) essential inputs ($\varkappa = \mathcal{S}$) or the remaining inputs ($\varkappa = \mathcal{N}$), respectively.

In Appendix B.6, we show that the partial derivative of industry j's outdegree (where j belongs to $\varkappa = \{\mathcal{S}, \mathcal{N}\}$) with respect to its price is given by:

$$\frac{\partial D_{jt}^{1,\text{out}}}{\partial P_{jt}} = \sum_{i=1}^{N} \Gamma_{ij}^{M} \left[1 - \varepsilon_{\varkappa} - (\varepsilon_{\mathcal{SN}} - \varepsilon_{\varkappa}) \frac{\omega_{ijt}}{\omega_{it}^{\varkappa}} - (1 - \varepsilon_{\mathcal{SN}})\omega_{ijt}\right] \left(\frac{P_{it}^{\varkappa}}{P_{jt}}\right)^{\varepsilon_{\varkappa}} \left(\frac{P_{it}^{M}}{P_{it}^{\varkappa}}\right)^{\varepsilon_{\mathcal{SN}}} \frac{1}{P_{it}^{M}}, \tag{B.39}$$

where $\omega_{it}^{\varkappa} = (P_{it}^{\varkappa} M_{it}^{\varkappa})/(P_{it}^{M} M_{it})$, i.e. the equilibrium share of expenses on the set of $\varkappa$ inputs in industry i's total intermediate consumption. In Appendix B.6, we show that if $\varepsilon_{\mathcal{SN}} < 1$—which seems plausible given our interpretation of the sets $\mathcal{S}$ and $\mathcal{N}$—the value of $\varepsilon_{\varkappa}$ that ensures that industry j's outdegree is increasing in its own price is given by:

$$\varepsilon_{\varkappa} < \frac{\omega_{it}^{\varkappa} - \omega_{ijt}\left[(1-\varepsilon_{\mathcal{SN}})\omega_{it}^{\varkappa} + \varepsilon_{\mathcal{SN}}\right]}{\omega_{it}^{\varkappa} - \omega_{ijt}}. \tag{B.40}$$

Note that the term $(1-\varepsilon_{\mathcal{SN}})\omega_{it}^{\varkappa} + \varepsilon_{\mathcal{SN}}$ is smaller than 1 if $\varepsilon_{\mathcal{SN}} < 1$, as we just assumed. The right-hand side in equation (B.40) is thus *larger* than 1. Therefore, if there exists a set of essential inputs alongside a set of remaining inputs, and if the elasticity of substitution between these two sets of inputs is less than unity, then the positive relationship between industries' outdegrees and their prices is conditional on the elasticity of substitution *within* the set they produce in being strictly less than the value on the right-hand side of (B.40)—which is a *weaker* requirement than that in equation (B.33), which required that this elasticity is less than 1.[17] In other words, to the extent that the intermediate aggregation process—featuring essential and less essential inputs—is more realistic than that in the baseline model, the condition that ensures that there exists a positive relationship between outdegrees and own prices is relatively *easier* to satisfy.[18]

B.6 Derivations for Appendix B.5 (Generalisation 2)

For ease of exposition (and without loss of generality for the sake of argument), assume that $\eta_i = 1$, i.e. all industries produce under constant returns to scale. Relative to the baseline model, we now have that the bundle of intermediate inputs consists of two sets of inputs, some (almost) essential and other which are non-essential, and is given by $M_{it} = \left[\delta_i^{\frac{1}{\varepsilon_{\mathcal{SN}}}}(M_{it}^{\mathcal{S}})^{\frac{\varepsilon_{\mathcal{SN}}-1}{\varepsilon_{\mathcal{SN}}}} + (1-\delta_i)^{\frac{1}{\varepsilon_{\mathcal{SN}}}}(M_{it}^{\mathcal{N}})^{\frac{\varepsilon_{\mathcal{SN}}-1}{\varepsilon_{\mathcal{SN}}}}\right]^{\frac{\varepsilon_{\mathcal{SN}}}{\varepsilon_{\mathcal{SN}}-1}}$. In turn, the bundle of essential inputs is given by $M_{it}^{\mathcal{S}} = \left[\sum_{j\in\mathcal{S}}\left(\tilde{\Gamma}_{ij}^{M}\right)^{\frac{1}{\varepsilon_{\mathcal{S}}}} M_{ijt}^{\frac{\varepsilon_{\mathcal{S}}-1}{\varepsilon_{\mathcal{S}}}}\right]^{\frac{\varepsilon_{\mathcal{S}}}{\varepsilon_{\mathcal{S}}-1}}$ and the non-essential bundle is given by $M_{it}^{\mathcal{N}} = \left[\sum_{j\in\mathcal{N}}\left(\tilde{\Gamma}_{ij}^{M}\right)^{\frac{1}{\varepsilon_{\mathcal{N}}}} M_{ijt}^{\frac{\varepsilon_{\mathcal{N}}-1}{\varepsilon_{\mathcal{N}}}}\right]^{\frac{\varepsilon_{\mathcal{N}}}{\varepsilon_{\mathcal{N}}-1}}$ where $\tilde{\Gamma}_{ij}^{M} = \Gamma_{ij}^{M}/\delta_i$ if $j\in\mathcal{S}$ and $\tilde{\Gamma}_{ij}^{M} = \Gamma_{ij}^{M}/(1-\delta_i)$ if $j\in\mathcal{N}$.

To derive the equation for intermediate input shares (and thus for industries' weighted outdegrees), one may follow the steps from Appendix B.3. The main difference is that, now, there is an FOC with respect to M_{ijt} if $j\in\mathcal{S}$, namely:

$$\begin{aligned} P_{jt} - \Lambda_{it}\Bigg[A_{it}L_{it}^{1-\mu_i}\left(\frac{\mu_i\varepsilon_{\mathcal{SN}}}{\varepsilon_{\mathcal{SN}}-1}M_{it}^{\frac{\mu_i\varepsilon_{\mathcal{SN}}-\varepsilon_{\mathcal{SN}}+1}{\varepsilon_{\mathcal{SN}}}}\delta_i^{\frac{1}{\varepsilon_{\mathcal{SN}}}}\frac{\varepsilon_{\mathcal{SN}}-1}{\varepsilon_{\mathcal{SN}}}(M_{it}^{\mathcal{S}})^{\frac{-1}{\varepsilon_{\mathcal{SN}}}}\right)& \\ \left(\frac{\varepsilon_{\mathcal{S}}}{\varepsilon_{\mathcal{S}}-1}(M_{it}^{\mathcal{S}})^{\frac{1}{\varepsilon_{\mathcal{S}}}}\left(\tilde{\Gamma}_{ij}^{M}\right)^{\frac{1}{\varepsilon_{\mathcal{S}}}}\frac{\varepsilon_{\mathcal{S}}-1}{\varepsilon_{\mathcal{S}}}M_{ijt}^{\frac{-1}{\varepsilon_{\mathcal{S}}}}\right)\Bigg] &= 0. \end{aligned} \tag{B.41}$$

[17] We clearly recover this condition if $\varepsilon_{\mathcal{SN}} = 1$.

[18] For example, if $\varepsilon_{\mathcal{SN}} = 0.1$, $\omega_{it}^{\varkappa} = 0.3$, and $\omega_{ijt} = 0.05$, then industry j's outdegree is increasing in its price (P_{jt}) as long as $\varepsilon_{\varkappa} < 1.166$.

Now, the natural price index for M_{it} is given by

$$P_{it}^{M}=\left[\delta_i(P_{it}^{\mathcal{S}})^{1-\varepsilon_{\mathcal{SN}}}+(1-\delta_i)(P_{it}^{\mathcal{N}})^{1-\varepsilon_{\mathcal{SN}}}\right]^{\frac{1}{1-\varepsilon_{\mathcal{SN}}}},$$

and the two subindices as

$$\begin{aligned}P_{it}^{\mathcal{S}}&=\left(\sum_{j\in\mathcal{S}}\tilde{\Gamma}_{ij}^{M}P_{jt}^{1-\varepsilon_{\mathcal{S}}}\right)^{\frac{1}{1-\varepsilon_{\mathcal{S}}}},\\ P_{it}^{\mathcal{N}}&=\left(\sum_{j\in\mathcal{N}}\tilde{\Gamma}_{ij}^{M}P_{jt}^{1-\varepsilon_{\mathcal{N}}}\right)^{\frac{1}{1-\varepsilon_{\mathcal{N}}}}.\end{aligned}$$

It can then be shown that

$$\begin{aligned}P_{it}^{\mathcal{S}}&=\Lambda_{it}\mu_iQ_{it}M_{it}^{\frac{1-\varepsilon_{\mathcal{SN}}}{\varepsilon_{\mathcal{SN}}}}\delta_i^{\frac{1}{\varepsilon_{\mathcal{SN}}}}(M_{it}^{\mathcal{S}})^{\frac{-1}{\varepsilon_{\mathcal{SN}}}},\\ P_{it}^{\mathcal{N}}&=\Lambda_{it}\mu_iQ_{it}M_{it}^{\frac{1-\varepsilon_{\mathcal{SN}}}{\varepsilon_{\mathcal{SN}}}}(1-\delta_i)^{\frac{1}{\varepsilon_{\mathcal{SN}}}}(M_{it}^{\mathcal{N}})^{\frac{-1}{\varepsilon_{\mathcal{SN}}}}.\end{aligned}$$

Following analogous steps to those in B.3, it can be easily shown that the intermediate input shares in equilibrium are now given by

$$\omega_{ijt}=\begin{cases}\Gamma_{ij}^{M}\left(\frac{P_{jt}}{P_{it}^{\mathcal{S}}}\right)^{1-\varepsilon_{\mathcal{S}}}\left(\frac{P_{it}^{\mathcal{S}}}{P_{it}^{M}}\right)^{1-\varepsilon_{\mathcal{SN}}} & \text{if } j\in\mathcal{S},\\ \Gamma_{ij}^{M}\left(\frac{P_{jt}}{P_{it}^{\mathcal{N}}}\right)^{1-\varepsilon_{\mathcal{N}}}\left(\frac{P_{it}^{\mathcal{N}}}{P_{it}^{M}}\right)^{1-\varepsilon_{\mathcal{SN}}} & \text{if } j\in\mathcal{N}.\end{cases}$$

Without loss of generality, let us focus on $j\in\mathcal{S}$. The first-order weighted outdegree of industry j is given by

$$D_{jt}^{1,\text{out}}:=\sum_{i=1}^{N}\omega_{ijt}=\sum_{i=1}^{N}\Gamma_{ij}^{M}\left(\frac{P_{jt}}{P_{it}^{\mathcal{S}}}\right)^{1-\varepsilon_{\mathcal{S}}}\left(\frac{P_{it}^{\mathcal{S}}}{P_{it}^{M}}\right)^{1-\varepsilon_{\mathcal{SN}}}.$$

Note that the partial derivatives of the price indices corresponding to essential and non-essential inputs with respect to P_{jt} are given by

$$\begin{aligned}\frac{\partial P_{it}^{\mathcal{S}}}{\partial P_{jt}}&=\frac{1}{1-\varepsilon_{\mathcal{S}}}\left[\sum_{j\in\mathcal{S}}\tilde{\Gamma}_{ij}^{M}P_{jt}^{1-\varepsilon_{\mathcal{S}}}\right]^{\frac{1-1+\varepsilon_{\mathcal{S}}}{1-\varepsilon_{\mathcal{S}}}}\tilde{\Gamma}_{ij}^{M}(1-\varepsilon_{\mathcal{S}})P_{jt}^{-\varepsilon_{\mathcal{S}}}\\ &=\tilde{\Gamma}_{ij}^{M}\left(\frac{P_{jt}}{P_{it}^{\mathcal{S}}}\right)^{-\varepsilon_{\mathcal{S}}},\end{aligned}$$

and

$$\frac{\partial P_{jt}^{M}}{\partial P_{jt}} = \frac{1}{1-\varepsilon_{\mathcal{SN}}}(P_{it}^{M})^{\varepsilon_{\mathcal{SN}}}\delta_i(1-\varepsilon_{\mathcal{SN}})(P_{it}^{\mathcal{S}})^{-\varepsilon_{\mathcal{SN}}}\frac{\partial P_{it}^{\mathcal{S}}}{\partial P_{jt}}$$
$$= \delta_i\left(\frac{P_{it}^{\mathcal{S}}}{P_{it}^{M}}\right)^{-\varepsilon_{\mathcal{SN}}}\tilde{\Gamma}_{ij}^{M}\left(\frac{P_{jt}}{P_{it}^{\mathcal{S}}}\right)^{-\varepsilon_{\mathcal{S}}}.$$

Therefore, the partial derivative of industry j's outdegree with respect to its own price is given by

$$\begin{aligned}
\frac{\partial D_{jt}^{1,\text{out}}}{\partial P_{jt}} = \sum_{i=1}^{N}\Bigg[&\Gamma_{ij}^{M}(1-\varepsilon_{\mathcal{S}})P_{jt}^{-\varepsilon_{\mathcal{S}}}(P_{it}^{\mathcal{S}})^{\varepsilon_{\mathcal{S}}-\varepsilon_{\mathcal{SN}}}(P_{it}^{M})^{\varepsilon_{\mathcal{SN}}-1}\\
&+\Gamma_{ij}^{M}P_{jt}^{1-\varepsilon_{\mathcal{S}}}(\varepsilon_{\mathcal{S}}-\varepsilon_{\mathcal{SN}})(P_{it}^{\mathcal{S}})^{\varepsilon_{\mathcal{S}}-\varepsilon_{\mathcal{SN}}-1}\frac{\partial P_{it}^{\mathcal{S}}}{\partial P_{jt}}(P_{it}^{M})^{\varepsilon_{\mathcal{SN}}-1}\\
&+\Gamma_{ij}^{M}P_{jt}^{1-\varepsilon_{\mathcal{S}}}(P_{it}^{\mathcal{S}})^{\varepsilon_{\mathcal{S}}-\varepsilon_{\mathcal{SN}}}(\varepsilon_{\mathcal{SN}}-1)(P_{it}^{M})^{\varepsilon_{\mathcal{SN}}-2}\frac{\partial P_{jt}^{M}}{\partial P_{jt}}\Bigg]\\
= \sum_{i=1}^{N}\Bigg[&\Gamma_{ij}^{M}(1-\varepsilon_{\mathcal{S}})P_{jt}^{-\varepsilon_{\mathcal{S}}}(P_{it}^{\mathcal{S}})^{\varepsilon_{\mathcal{S}}-\varepsilon_{\mathcal{SN}}}(P_{it}^{M})^{\varepsilon_{\mathcal{SN}}-1}\\
&+\Gamma_{ij}^{M}P_{jt}^{1-\varepsilon_{\mathcal{S}}}(\varepsilon_{\mathcal{S}}-\varepsilon_{\mathcal{SN}})(P_{it}^{\mathcal{S}})^{\varepsilon_{\mathcal{S}}-\varepsilon_{\mathcal{SN}}-1}\tilde{\Gamma}_{ij}^{M}\left(\frac{P_{jt}}{P_{it}^{\mathcal{S}}}\right)^{-\varepsilon_{\mathcal{S}}}(P_{it}^{M})^{\varepsilon_{\mathcal{SN}}-1}\\
&+\Gamma_{ij}^{M}P_{jt}^{1-\varepsilon_{\mathcal{S}}}(P_{it}^{\mathcal{S}})^{\varepsilon_{\mathcal{S}}-\varepsilon_{\mathcal{SN}}}(\varepsilon_{\mathcal{SN}}-1)(P_{it}^{M})^{\varepsilon_{\mathcal{SN}}-2}\delta_i\left(\frac{P_{it}^{\mathcal{S}}}{P_{it}^{M}}\right)^{-\varepsilon_{\mathcal{SN}}}\tilde{\Gamma}_{ij}^{M}\left(\frac{P_{jt}}{P_{it}^{\mathcal{S}}}\right)^{-\varepsilon_{\mathcal{S}}}\Bigg].
\end{aligned}$$

Taking out the first term inside the square brackets, we can rewrite this as

$$\begin{aligned}
\frac{\partial D_{jt}^{1,\text{out}}}{\partial P_{jt}} = \sum_{i=1}^{N}\Bigg[&\Gamma_{ij}^{M}(1-\varepsilon_{\mathcal{S}})P_{jt}^{-\varepsilon_{\mathcal{S}}}(P_{it}^{\mathcal{S}})^{\varepsilon_{\mathcal{S}}-\varepsilon_{\mathcal{SN}}}(P_{it}^{M})^{\varepsilon_{\mathcal{SN}}-1}\\
&\Bigg(1+\frac{\varepsilon_{\mathcal{S}}-\varepsilon_{\mathcal{SN}}}{1-\varepsilon_{\mathcal{S}}}P_{jt}(P_{it}^{\mathcal{S}})^{-1}\tilde{\Gamma}_{ij}^{M}\left(\frac{P_{jt}}{P_{it}^{\mathcal{S}}}\right)^{-\varepsilon_{\mathcal{S}}}\\
&+\frac{\varepsilon_{\mathcal{SN}}-1}{1-\varepsilon_{\mathcal{S}}}P_{jt}(P_{it}^{M})^{-1}\delta_i\left(\frac{P_{it}^{\mathcal{S}}}{P_{it}^{M}}\right)^{-\varepsilon_{\mathcal{SN}}}\tilde{\Gamma}_{ij}^{M}\left(\frac{P_{jt}}{P_{it}^{\mathcal{S}}}\right)^{-\varepsilon_{\mathcal{S}}}\Bigg)\Bigg].
\end{aligned}$$

Assuming $\varepsilon_{\mathcal{S}} < 1$, i.e. the essential inputs are relative complements, the sign of $\partial D_{jt}^{1,\text{out}}/\partial P_{jt}$ is thus determined by the sign of the term multiplying it. Define $\omega_{it}^{\mathcal{S}}$ as the share of expenditure on an essential input i in industry i's total expenditure on intermediate inputs, i.e.,

$$
\begin{aligned}
\omega_{it}^{\mathcal{S}} &= \frac{P_{it}^{\mathcal{S}} M_{it}^{\mathcal{S}}}{P_{it}^{M} M_{it}} \\
&= \frac{P_{it}^{\mathcal{S}} \delta_i M_{it} (P_{it}^{\mathcal{S}})^{-\varepsilon_{\mathcal{SN}}} (P_{it}^{M})^{\varepsilon_{\mathcal{SN}}}}{P_{it}^{M} M_{it}} \\
&= \delta_i \left(\frac{P_{it}^{\mathcal{S}}}{P_{it}^{M}} \right)^{1-\varepsilon_{\mathcal{SN}}}.
\end{aligned}
$$

Note that

$$
\begin{aligned}
P_{jt} (P_{it}^{\mathcal{S}})^{-1} \tilde{\Gamma}_{ij}^{M} \left(\frac{P_{jt}}{P_{it}^{\mathcal{S}}} \right)^{-\varepsilon_{\mathcal{S}}} &= \frac{P_{jt}^{1-\varepsilon_{\mathcal{S}}}}{(P_{it}^{\mathcal{S}})^{1-\varepsilon_{\mathcal{S}}}} \tilde{\Gamma}_{ij}^{M} \\
&= \omega_{ijt} \delta_i^{-1} \left(\frac{P_{it}^{\mathcal{S}}}{P_{it}^{M}} \right)^{\varepsilon_{\mathcal{SN}}-1} \\
&= \frac{\omega_{ijt}}{\omega_{it}^{\mathcal{S}}}.
\end{aligned}
$$

Note also that

$$
\begin{aligned}
P_{jt} (P_{it}^{M})^{-1} \delta_i \left(\frac{P_{it}^{\mathcal{S}}}{P_{it}^{M}} \right)^{-\varepsilon_{\mathcal{SN}}} \tilde{\Gamma}_{ij}^{M} \left(\frac{P_{jt}}{P_{it}^{\mathcal{S}}} \right)^{-\varepsilon_{\mathcal{S}}} &= \frac{\omega_{ijt}}{\omega_{it}^{\mathcal{S}}} (P_{it}^{\mathcal{S}}) (P_{it}^{M})^{-1} \delta_i \left(\frac{P_{it}^{\mathcal{S}}}{P_{it}^{M}} \right)^{-\varepsilon_{\mathcal{SN}}} \\
&= \frac{\omega_{ijt}}{\omega_{it}^{\mathcal{S}}} \omega_{it}^{\mathcal{S}} = \omega_{ijt}.
\end{aligned}
$$

Therefore, we have

$$
\begin{aligned}
\frac{\partial D_{jt}^{1,\text{out}}}{\partial P_{jt}} &= \sum_{i=1}^{N} \left[\Gamma_{ij}^{M} (1-\varepsilon_{\mathcal{S}}) P_{jt}^{-\varepsilon_{\mathcal{S}}} (P_{it}^{\mathcal{S}})^{\varepsilon_{\mathcal{S}}-\varepsilon_{\mathcal{SN}}} (P_{it}^{M})^{\varepsilon_{\mathcal{SN}}-1} \left(1 + \frac{\varepsilon_{\mathcal{S}} - \varepsilon_{\mathcal{SN}}}{1-\varepsilon_{\mathcal{S}}} \frac{\omega_{ijt}}{\omega_{it}^{\mathcal{S}}} + \frac{\varepsilon_{\mathcal{SN}} - 1}{1-\varepsilon_{\mathcal{S}}} \omega_{ijt} \right) \right] \\
&= \sum_{i=1}^{N} \left[\Gamma_{ij}^{M} P_{jt}^{-\varepsilon_{\mathcal{S}}} (P_{it}^{\mathcal{S}})^{\varepsilon_{\mathcal{S}}-\varepsilon_{\mathcal{SN}}} (P_{it}^{M})^{\varepsilon_{\mathcal{SN}}-1} \left(1 - \varepsilon_{\mathcal{S}} - (\varepsilon_{\mathcal{SN}} - \varepsilon_{\mathcal{S}}) \frac{\omega_{ijt}}{\omega_{it}^{\mathcal{S}}} - (1-\varepsilon_{\mathcal{SN}}) \omega_{ijt} \right) \right].
\end{aligned}
\tag{B.42}
$$

If $\varepsilon_{\mathcal{S}} = \varepsilon_{\mathcal{SN}}$, we recover the expression for $\partial D_{jt}^{1,\text{out}} / \partial P_{jt}$ from our baseline model in which the elasticity of substitution across intermediate inputs is the same for all inputs. If $\varepsilon_{\mathcal{S}} \neq \varepsilon_{\mathcal{SN}}$, then a sufficient condition for industry j's first-order weighted outdegree to be increasing in its price is:

$$
1 - \varepsilon_{\mathcal{S}} - (\varepsilon_{\mathcal{SN}} - \varepsilon_{\mathcal{S}}) \frac{\omega_{ijt}}{\omega_{it}^{\mathcal{S}}} - (1-\varepsilon_{\mathcal{SN}}) \omega_{ijt} > 0.
$$

We can rewrite this inequality as

$$
(1-\varepsilon_{\mathcal{S}}) \omega_{it}^{\mathcal{S}} - (\varepsilon_{\mathcal{SN}} - \varepsilon_{\mathcal{S}}) \omega_{ijt} - (1-\varepsilon_{\mathcal{SN}}) \omega_{ijt} \omega_{it}^{\mathcal{S}} > 0.
$$

Collecting the terms involving $\varepsilon_{\mathcal{S}}$ on the left-hand side, we have

$$\left(\omega_{ijt} - \omega_{it}^{\mathcal{S}}\right) \varepsilon_{\mathcal{S}} > (1 - \varepsilon_{\mathcal{SN}})\omega_{ijt}\omega_{it}^{\mathcal{S}} - \omega_{it}^{\mathcal{S}} + \varepsilon_{\mathcal{SN}}\omega_{ijt}.$$

As long as $\omega_{it}^{\mathcal{S}} > \omega_{ijt}$, i.e. that the share of expenditure on essential inputs in total intermediate consumption exceeds the share of an individual good j in total intermediate consumption, we have that industry j's outdegree is increasing in its price if

$$\varepsilon_{\mathcal{S}} < \frac{\omega_{it}^{\mathcal{S}} - \omega_{ijt}\left[(1 - \varepsilon_{\mathcal{SN}})\omega_{it}^{\mathcal{S}} + \varepsilon_{\mathcal{SN}}\right]}{\omega_{it}^{\mathcal{S}} - \omega_{ijt}}. \tag{B.43}$$

Consider the following three (exhaustive) possibilities. First, the right-hand side of equation (B.43) equals one if $\varepsilon_{\mathcal{SN}} = 1$. Second, it will be greater than one if $(1 - \varepsilon_{\mathcal{SN}})\omega_{it}^{\mathcal{S}} + \varepsilon_{\mathcal{SN}} < 1$. Note that this condition can be equivalently written as $\varepsilon_{\mathcal{SN}}\left(1 - \omega_{it}^{\mathcal{S}}\right) < 1 - \omega_{it}^{\mathcal{S}}$, which simplifies to the requirement that $\varepsilon_{\mathcal{SN}} < 1$. Third, the right-hand side of equation (B.43) will be smaller than one if $(1 - \varepsilon_{\mathcal{SN}})\omega_{it}^{\mathcal{S}} + \varepsilon_{\mathcal{SN}} > 1$. This condition similarly simplifies to a requirement that $\varepsilon_{\mathcal{SN}} > 1$.

Recall that we assumed earlier that $\varepsilon_{\mathcal{S}} < 1$. Since one may reasonably expect that $\varepsilon_{\mathcal{SN}}$ will also be smaller than one in practice—implying that the sets of essential and non-essential inputs are relative complements—we have that the sufficient condition to ensure that industry j's outdegree is increasing in its price is given by

$$\varepsilon_{\mathcal{S}} < \frac{\omega_{ijt}\left[(1 - \varepsilon_{\mathcal{SN}})\omega_{it}^{\mathcal{S}} + \varepsilon_{\mathcal{SN}}\right] - \omega_{it}^{\mathcal{S}}}{\omega_{ijt} - \omega_{it}^{\mathcal{S}}}, \tag{B.44}$$

where the right-hand side of inequality (B.43) is larger than one. In other words, if the 'between-elasticity' ($\varepsilon_{\mathcal{SN}}$) is less than 1, then the within-elasticity ($\varepsilon_{\mathcal{S}}$) may exceed unity but must not be larger than the right-hand-side expression in inequality (B.44) in order to ensure that industries' first-order weighted outdegrees increase with own prices.

Recall that in deriving the condition given by (B.43), we focused on the case $j \in \mathcal{S}$. This was without loss of generality and an analogous condition applies for $j \in \mathcal{N}$.

C. Derivations Related to Section 3

Throughout this section, matrices are denoted in bold. The setup follows Section 3.1 of the main text. We derive the non-stochastic steady state in Section C.1, the log-linearised equilibrium in Section C.2, and the components of the model filter in Sections C.3–C.6.

C.1 Solving for the Non-Stochastic Steady State

Our model economy is identical to that in Atalay (2017) *except* that we allow for there to (potentially) be non-constant returns to scale. In other words, we allow the values of η_i to be potentially different from one. As a result, the steady state of the economy in Section 3 will potentially be different from that in Atalay (2017) even if all the other parameters had identical values. In solving for the steady state, we will make extensive references to Appendix F in Atalay (2017).

First, we want to show how the cost-minimising solutions for intermediate inputs and the capital-labour bundle change under non-constant returns to scale. The price index of a bundle of intermediate goods in industry i is given by $P_{it}^M = \left[\sum_{j=1}^N \Gamma_{ij}^M P_{jt}^{1-\varepsilon_M}\right]^{\frac{1}{1-\varepsilon_M}}$. Using the first-order condition with respect to M_{ij}, we have that with a potentially non-unitary η_i (assuming that the steady-state values of technology are $A_i = 1$ for all i):

$$\frac{P_j^{1-\varepsilon_M}}{P_i^{1-\varepsilon_M}} = \left(\frac{\mu_i}{M_i}\right)^{\frac{1-\varepsilon_M}{\varepsilon_Q}} \left(\frac{M_i \Gamma_{ij}^M}{M_{ij}}\right)^{\frac{1-\varepsilon_M}{\varepsilon_M}} \eta_i^{1-\varepsilon_M} Q_i^{(1-\varepsilon_M)\frac{\varepsilon_Q(\eta_i-1)+1}{\eta_i\varepsilon_Q}}.$$

We can rearrange this equation so as to have P_i^M on one side, obtaining:

$$P_i^M = P_i \left(\frac{\mu_i}{M_i}\right)^{\frac{1}{\varepsilon_Q}} M_i^{\frac{1}{\varepsilon_M}} \eta_i Q_i^{\frac{\varepsilon_Q(\eta_i-1)+1}{\eta_i\varepsilon_Q}} \left[\sum_{j=1}^N \Gamma_{ij}^M \left(\frac{\Gamma_{ij}^M}{M_{ij}}\right)^{\frac{1-\varepsilon_M}{\varepsilon_M}}\right]^{\frac{1}{1-\varepsilon_M}}.$$

Note that, by definition, $M_i = \left[\sum_{j=1}^N (\Gamma_{ij}^M)^{\frac{1}{\varepsilon_M}} (M_{ij})^{\frac{\varepsilon_M-1}{\varepsilon_M}}\right]^{\frac{\varepsilon_M}{\varepsilon_M-1}}$. The above equation can thus be simplified to:

$$\left(\frac{P_i^M}{P_i}\right)^{1-\varepsilon_Q} = \left(\frac{\mu_i}{M_i}\right)^{\frac{1-\varepsilon_Q}{\varepsilon_Q}} \eta_i^{1-\varepsilon_Q} Q_i^{(1-\varepsilon_Q)\frac{\varepsilon_Q(\eta_i-1)+1}{\eta_i\varepsilon_Q}}. \tag{C.1}$$

If we have constant returns to scale (i.e. $\eta_i = 1$), then the above equation simplifies to:

$$\left(\frac{P_i^M}{P_i}\right)^{1-\varepsilon_Q} = \left(\frac{\mu_i}{M_i}\right)^{\frac{1-\varepsilon_Q}{\varepsilon_Q}} Q_i^{\frac{1-\varepsilon_Q}{\varepsilon_Q}}$$

which corresponds to equation (35) in Appendix F of Atalay (2017). But with a non-unitary η_i, we have that:

$$\left(\frac{P_i^M}{P_i}\right)^{1-\varepsilon_Q} \mu_i \eta_i^{\varepsilon_Q - 1} Q_i^{(\varepsilon_Q(\eta_i - 1)+1)\frac{(\varepsilon_Q-1)}{\eta_i \varepsilon_Q}} = \mu_i^{\frac{1}{\varepsilon_Q}} M_i^{\frac{\varepsilon_Q - 1}{\varepsilon_Q}}. \tag{C.2}$$

The cost-minimising choice for the capital-labour bundle will also depend on η_i and is given by:

$$(1-\mu_i)^{\frac{1}{\varepsilon_Q}} \left(\left(\frac{K_i}{\alpha_i}\right)^{\alpha_i}\left(\frac{L_i}{1-\alpha_i}\right)^{1-\alpha_i}\right)^{\frac{\varepsilon_Q-1}{\varepsilon_Q}} = \eta_i^{\varepsilon_Q-1}(1-\mu_i) Q_i^{(\varepsilon_Q(\eta_i-1)+1)\frac{(\varepsilon_Q-1)}{\eta_i\varepsilon_Q}} \times \left(\frac{\left(\frac{1-\beta(1-\delta_K)}{\beta}\right)^{\alpha_i}\left[\sum_{j=1}^N \Gamma_{ij}^X P_j^{1-\varepsilon_X}\right]^{\frac{\alpha_i}{1-\varepsilon_X}}}{P_i}\right)^{1-\varepsilon_Q}. \tag{C.3}$$

Next, we substitute the cost-minimising solutions for the capital-labour bundle and intermediate inputs—given by equations (C.2) and (C.3)—into the production function, which at steady state is given by:

$$Q_i = \left[(1-\mu_i)^{\frac{1}{\varepsilon_Q}} \left(\left(\frac{K_i}{\alpha_i}\right)^{\alpha_i}\left(\frac{L_i}{1-\alpha_i}\right)^{1-\alpha_i}\right)^{\frac{\varepsilon_Q-1}{\varepsilon_Q}} + \mu_i^{\frac{1}{\varepsilon_Q}} (M_i)^{\frac{\varepsilon_Q-1}{\varepsilon_Q}}\right]^{\eta_i \frac{\varepsilon_Q}{\varepsilon_Q-1}},$$

to obtain:

$$Q_i = \left[\eta_i^{\varepsilon_Q-1}(1-\mu_i) Q_i^{(\varepsilon_Q(\eta_i-1)+1)\frac{(\varepsilon_Q-1)}{\eta_i\varepsilon_Q}} \left(\frac{\left(\frac{1-\beta(1-\delta_K)}{\beta}\right)^{\alpha_i}\left[\sum_{j=1}^N \Gamma_{ij}^X P_j^{1-\varepsilon_X}\right]^{\frac{\alpha_i}{1-\varepsilon_X}}}{P_i}\right)^{1-\varepsilon_Q} + \left(\frac{P_i^M}{P_i}\right)^{1-\varepsilon_Q} \mu_i \eta_i^{\varepsilon_Q-1} Q_i^{(\varepsilon_Q(\eta_i-1)+1)\frac{(\varepsilon_Q-1)}{\eta_i\varepsilon_Q}}\right]^{\frac{\eta_i\varepsilon_Q}{\varepsilon_Q-1}}.$$

We can see that Q_i will drop out of the above equation under constant returns to scale, but not under non-constant returns to scale. We can simplify the above equation to obtain:

$$Q_i^{\frac{\varepsilon_Q - 1}{\eta_i \varepsilon_Q}\left(1-(\varepsilon_Q(\eta_i-1)+1)\right)} = \eta_i^{\varepsilon_Q-1}(1-\mu_i)\left(\frac{\left(\frac{1-\beta(1-\delta_K)}{\beta}\right)^{\alpha_i}\left[\sum_{j=1}^{N}\Gamma_{ij}^{X}P_j^{1-\varepsilon_X}\right]^{\frac{\alpha_i}{1-\varepsilon_X}}}{P_i}\right)^{1-\varepsilon_Q} + \left(\frac{P_i^M}{P_i}\right)^{1-\varepsilon_Q}\mu_i\eta_i^{\varepsilon_Q-1}. \quad \text{(C.4)}$$

Before turning to the goods-market clearing condition, it is convenient to record the profit identity that follows from cost minimisation. At the cost-minimising allocation, total expenditure on the value-added bundle and on intermediates can be combined to yield $W_t L_{it} + r_t^K K_{it} + P_{it}^M M_{it} = \eta_i \Lambda_{it} Q_{it}$, where Λ_{it} is the Lagrange multiplier on the production constraint. Combined with $P_{it} = \Lambda_{it}$ from perfect competition, the profit of industry i is therefore

$$\Pi_{it} \;\coloneqq\; P_{it}Q_{it} - W_t L_{it} - r_t^K K_{it} - P_{it}^M M_{it} \;=\; (1-\eta_i)\,P_{it}Q_{it}. \quad \text{(C.5)}$$

Profits in industry i are thus a fraction $(1-\eta_i)$ of nominal gross output—positive whenever returns to scale are decreasing, and zero in the constant-returns-to-scale limit. Aggregating, $\sum_{i=1}^{N}\Pi_{it} = \sum_{i=1}^{N}(1-\eta_i)P_{it}Q_{it}$, which is the profit term that enters the household's budget constraint. Throughout the rest of this section, the η_i factors that appear in the cost-minimising solutions and in the steady-state shares (Section C.1) embed this profit-distribution structure; under constant returns to scale, profits vanish and the closed-economy resource constraint reduces to the form used by Atalay (2017).

Equation (C.4) gives us a set of N equations in prices and quantities, and we need another set of N equations in order to solve for the steady-state prices and quantities. As in the simplified model, we make use of the goods-market clearing condition. The market clearing condition for good j is given by:

$$Q_j = \delta_{C_j}C_j + \sum_{i=1}^{N}(M_{ij} + X_{ij}). \quad \text{(C.6)}$$

We thus need to write the right-hand side of the above equation in terms of prices and quantities. The consumption of good j will still be given by equation (39) in Appendix F of Atalay (2017). The solution for M_{ij} will, however, depend on η_i, and we make use of the first-order condition with respect to it, which is given by:

$$M_{ij} = (\mu_i)^{\frac{\varepsilon_M}{\varepsilon_Q}}(M_i)^{\frac{\varepsilon_Q-\varepsilon_M}{\varepsilon_Q}}\Gamma_{ij}^{M}\left(\frac{P_i}{P_j}\right)^{\varepsilon_M}\left(\eta_i Q_i^{\frac{\varepsilon_Q(\eta_i-1)+1}{\eta_i\varepsilon_Q}}\right)^{\varepsilon_M}. \quad \text{(C.7)}$$

Using equation (C.2), we can substitute in for M_i since:

$$M_i = \left(\frac{P_i^M}{P_i}\right)^{-\varepsilon_Q} \mu_i \eta_i^{\varepsilon_Q} Q_i^{\frac{\varepsilon_Q(\eta_i-1)+1}{\eta_i}}.$$

We thus have that:

$$M_{ij} = \left(\eta_i Q_i^{\frac{\varepsilon_Q(\eta_i-1)+1}{\eta_i \varepsilon_Q}}\right)^{\varepsilon_Q} \mu_i \Gamma_{ij}^M P_j^{-\varepsilon_M} (P_i^M)^{\varepsilon_M - \varepsilon_Q} P_i^{\varepsilon_Q}.$$

Next, we solve for the investment input purchases by industry i sourced from industry j. The cost-minimising optimality condition for capital, which equates the rental price of a unit of capital to its marginal revenue product, is given by:

$$\begin{aligned}\frac{1-\beta(1-\delta_K)}{\beta}\left[\sum_{j=1}^{N}\Gamma_{ij}^X P_j^{1-\varepsilon_X}\right]^{\frac{1}{1-\varepsilon_X}} &= P_i(1-\mu_i)^{\frac{1}{\varepsilon_Q}} \eta_i Q_i^{\frac{\varepsilon_Q(\eta_i-1)+1}{\eta_i \varepsilon_Q}} \\ &\quad \cdot \left(\frac{K_i}{\alpha_i}\right)^{\alpha_i \frac{\varepsilon_Q - 1}{\varepsilon_Q} - 1} \left(\frac{L_i}{1-\alpha_i}\right)^{(1-\alpha_i)\frac{\varepsilon_Q-1}{\varepsilon_Q}}.\end{aligned}$$

Using the cost-minimising solution for the capital-labour bundle given by equation (C.3), the above equation can be simplified to yield:

$$\left(\frac{K_i}{\alpha_i}\right) = \left[\frac{1-\beta(1-\delta_K)}{\beta}\left(\sum_{j=1}^{N}\Gamma_{ij}^X P_j^{1-\varepsilon_X}\right)^{\frac{1}{1-\varepsilon_X}}\right]^{-1+\alpha_i(1-\varepsilon_Q)} \eta_i^{\varepsilon_Q} Q_i^{\frac{\varepsilon_Q(\eta_i-1)+1}{\eta_i}} (1-\mu_i)(P_i)^{\varepsilon_Q}. \tag{C.8}$$

Recall that $X_i = \delta_K K_i$. Using equation (C.8), we thus have that the cost-minimising solution for the investment input purchases of industry i from industry j is given by:

$$\begin{aligned} X_{ij} &= X_i \Gamma_{ij}^X \left(\frac{P_j}{P_i^X}\right)^{-\varepsilon_X} \\ &= X_i \Gamma_{ij}^X (P_j)^{-\varepsilon_X} (P_i^X)^{\varepsilon_X} \\ &= \Gamma_{ij}^X (P_j)^{-\varepsilon_X} (1-\mu_i) \eta_i^{\varepsilon_Q} Q_i^{\frac{\varepsilon_Q(\eta_i-1)+1}{\eta_i}} \alpha_i \delta_K \\ &\quad \cdot \left(\frac{1-\beta(1-\delta_K)}{\beta}\right)^{-1+\alpha_i(1-\varepsilon_Q)} \left[\sum_{j'=1}^{N}\Gamma_{ij'}^X P_{j'}^{1-\varepsilon_X}\right]^{\frac{\varepsilon_X - 1 + \alpha_i(1-\varepsilon_Q)}{1-\varepsilon_X}} (P_i)^{\varepsilon_Q}. \end{aligned} \tag{C.9}$$

Finally, we can substitute the solutions for C_j, M_{ij}, and X_{ij} (given by equation (39) in Appendix F of Atalay (2017), equation (C.2), and equation (C.9), respectively) into the market clearing condition for each good j (given by equation (C.6)) to obtain:

$$Q_j - \sum_{i=1}^{N} \tilde{\Gamma}_{ji} \eta_i^{\varepsilon_Q} Q_i^{\frac{\varepsilon_Q(\eta_i - 1)+1}{\eta_i}} = \xi_j (\delta_{C_j})^{\varepsilon_D} \left(\frac{1 - \beta(1-\delta_{C_j})}{\beta} \right)^{-\varepsilon_D} (P_j)^{-\varepsilon_D} \bar{C}^{1-\varepsilon_D}, \tag{C.10}$$

where $\tilde{\Gamma}_{ji}$ is an auxiliary coefficient (indexed supplier-first) that absorbs the Γ_{ij}^M and Γ_{ij}^X contributions arising in M_{ij} and X_{ij}. This equation, together with the set of N equations given by (C.4), yields a system of $2N$ equations in $2N$ unknowns, Q_j and P_j for all $j \in \{1, \dots, N\}$, which we can solve using linear algebra. More specifically, we solve this system as follows. First, we use equation (C.4) to express Q_j as a function of prices only. We then substitute for quantities in equation (C.10) and solve the resulting system of N equations in N unknowns.

In terms of the steady-state shares derived on page 55 in Appendix F of Atalay (2017), introducing potentially non-constant returns to scale implies that the first equation (equation (42) in Atalay's (2017) Appendix F) will change. To see why, start from the FOC w.r.t. L_i and substitute in the cost-minimising solution for the capital-labour bundle to get:

$$1 = P_i^{\varepsilon_Q} \eta_i^{\varepsilon_Q} (1-\mu_i) Q_i^{\frac{\varepsilon_Q(\eta_i-1)+1}{\eta_i}} \left[\left(\frac{1-\beta(1-\delta_K)}{\beta} \right) P_i^X \right]^{\alpha_i(1-\varepsilon_Q)}.$$

The next two equations (with C_i and S_i^C on the left-hand side, respectively) are invariant to the degree of returns to scale. The subsequent equation, with M_{ij}/Q_j on the left-hand side, will change to:

$$\frac{M_{ij}}{Q_j} = \frac{1}{Q_j} \eta_i^{\varepsilon_Q} Q_i^{\frac{\varepsilon_Q(\eta_i-1)+1}{\eta_i}} \mu_i \Gamma_{ij}^M P_j^{-\varepsilon_M} (P_i^M)^{\varepsilon_M - \varepsilon_Q} P_i^{\varepsilon_Q}.$$

The equation with X_{ij}/Q_j on the left-hand side will similarly change, with Q_i in the equation Atalay (2017) derives replaced by $\eta_i^{\varepsilon_Q} Q_i^{\frac{\varepsilon_Q(\eta_i-1)+1}{\eta_i}}$. Finally, equations (44) and (45) on page 55 of Atalay (2017) will remain unchanged regardless of whether $\eta_i = 1$ for all i.

C.2 Log-Linearisation of the Model Equilibrium

We now log-linearise the model equilibrium conditions. Relaxing the assumption of constant returns to scale will imply the following changes to the log-linear equations derived in Appendix F.2 in Atalay (2017). First, the equation for the relative price between industries j and i will change to:

$$\hat{p}_{jt} - \hat{p}_{it} = \frac{\varepsilon_Q - 1}{\varepsilon_Q}\hat{a}_{it} + \frac{\varepsilon_Q(\eta_i - 1) + 1}{\eta_i \varepsilon_Q}\hat{q}_{it} + \left(\frac{1}{\varepsilon_M} - \frac{1}{\varepsilon_Q}\right)\hat{m}_{it} - \frac{1}{\varepsilon_M}\hat{m}_{ijt},$$

i.e. the coefficient multiplying $\hat{q}_{it}$ will change. Second, the equation for labour will change analogously, with the coefficient multiplying $\hat{q}_{it}$ changing from $\frac{1}{\varepsilon_Q}$ to $\frac{\varepsilon_Q(\eta_i-1)+1}{\eta_i\varepsilon_Q}$:

$$\begin{aligned}\frac{1}{\varepsilon_{LS}}\sum_{i'=1}^{N} S^L_{i'}\hat{l}_{i't} = \hat{p}_{it} + \frac{\varepsilon_Q - 1}{\varepsilon_Q}\hat{a}_{it} + \frac{\varepsilon_Q(\eta_i - 1) + 1}{\eta_i \varepsilon_Q}\hat{q}_{it} \\ + \alpha_i\frac{\varepsilon_Q - 1}{\varepsilon_Q}\hat{k}_{it} + \frac{\alpha_i - 1 - \alpha_i\varepsilon_Q}{\varepsilon_Q}\hat{l}_{it}.\end{aligned}$$

Third, the log-linearised FOC w.r.t. $K_{i,t+1}$ will change similarly to:

$$\begin{aligned}\frac{1}{1-\beta(1-\delta_K)}\hat{p}^X_{it} - \frac{\beta(1-\delta_K)}{1-\beta(1-\delta_K)}\hat{p}^X_{i,t+1} = \hat{p}_{i,t+1} + \frac{\varepsilon_Q(\eta_i - 1) + 1}{\eta_i \varepsilon_Q}\hat{q}_{i,t+1} \\ + \frac{\varepsilon_Q - 1}{\varepsilon_Q}\hat{a}_{i,t+1} \\ + \frac{(\varepsilon_Q - 1)(1-\alpha_i)}{\varepsilon_Q}\hat{l}_{i,t+1} + \left[-1 + \frac{\alpha_i(\varepsilon_Q - 1)}{\varepsilon_Q}\right]\hat{k}_{i,t+1}.\end{aligned}$$

Finally, the log-linearised form of the production function will change to:

$$\hat{q}_{it} = \eta_i\left[\hat{a}_{it} + \alpha_i(1 - S_{M,i})\hat{k}_{it} + (1-\alpha_i)(1-S_{M,i})\hat{l}_{it} + S_{M,i}\hat{m}_{it}\right].$$

The remaining equations shown on page 56 in Appendix F.2 of Atalay (2017) will remain unchanged.

Accordingly, the bottom four equations on page 57 of Appendix F.2 of Atalay (2017) will change. Let Σ denote diag(η), where η denotes the $N \times 1$ vector of η_i's. Then, in the equation with $\hat{m}_t$ on the left-hand side, the coefficient on $\hat{q}_t$ on the right-hand side will change to $\frac{\varepsilon_M}{\varepsilon_Q}\left[\varepsilon_Q\left(\Sigma - \mathbf{I}\right) + \mathbf{I}\right]\Sigma^{-1}T_2$. Similarly, in the equations with $\frac{1}{\varepsilon_{LS}}S^L\hat{l}_t$ and $\hat{p}^X_t$ on the left-hand side, the coefficient on $\hat{q}_t$ on the right-hand side will change to $\frac{1}{\varepsilon_Q}\left[\varepsilon_Q\left(\Sigma - \mathbf{I}\right) + \mathbf{I}\right]\Sigma^{-1}$. Finally, Σ will premultiply all terms on the right-hand side in the last equation (corresponding to the log-linearised production function in matrix form):

$$\hat{q}_t = \Sigma\left[\hat{a}_t + \alpha(\mathbf{I} - S_M)\hat{k}_t + (\mathbf{I} - \alpha)(\mathbf{I} - S_M)\hat{l}_t + S_M\hat{\mathbb{M}}_t\right],$$

where $\hat{\mathbb{M}}_t = (\varepsilon_Q - 1)\hat{a}_t + [\varepsilon_Q(\Sigma - \mathbf{I}) + \mathbf{I}]\,\Sigma^{-1}\hat{q}_t + \varepsilon_Q(\mathbf{I} - S_1^M)\hat{p}_t$. Substituting in for $\hat{\mathbb{M}}_t$, collecting the terms involving $\hat{q}_t$ on the left-hand side, and solving for $\hat{q}_t$ yields:

$$\hat{q}_t = \left[\Sigma^{-1} - S_M\left[\varepsilon_Q(\Sigma - \mathbf{I}) + \mathbf{I}\right]\Sigma^{-1}\right]^{-1}\Bigg[(\mathbf{I} + S_M(\varepsilon_Q - 1))\hat{a}_t$$
$$+ \alpha(\mathbf{I} - S_M)\hat{k}_t + (\mathbf{I} - \alpha)(\mathbf{I} - S_M)\hat{l}_t + S_M\varepsilon_Q(\mathbf{I} - S_1^M)\hat{p}_t\Bigg]. \tag{C.11}$$

We can then plug the above equation for $\hat{q}_t$ into the remaining log-linear equations shown in *Step 3* on page 59 in Atalay's (2017) Appendix F. First, we have that in the equation involving $\hat{c}_t$ and $\hat{c}_{t+1}$ (inter alia):

$$\begin{aligned}
0 &= \delta_C^{-1}\tilde{S}_C^Q\hat{c}_{t+1} + (\mathbf{I} - \delta_C^{-1})\tilde{S}_C^Q\hat{c}_t + (\varepsilon_Q - 1)\tilde{S}_M^Q T_2\hat{a}_t + \tilde{S}_X^Q T_2\delta_K^{-1}\hat{k}_{t+1} + \tilde{S}_X^Q T_2(1 - \delta_K^{-1})\hat{k}_t \\
&+ \left[\varepsilon_Q\tilde{S}_M^Q T_2(\mathbf{I} - S_1^M) + \varepsilon_M\tilde{S}_M^Q[T_2S_1^M - T_1] + \varepsilon_X\tilde{S}_X^Q[T_2S_1^X - T_1]\right]\hat{p}_t \\
&- \left(\mathbf{I} - \tilde{S}_M^Q T_2\right)\left[\Sigma^{-1} - S_M\left[\varepsilon_Q(\Sigma - \mathbf{I}) + \mathbf{I}\right]\Sigma^{-1}\right]^{-1}\Bigg[(\mathbf{I} + S_M(\varepsilon_Q - 1))\hat{a}_t \\
&+ \alpha(\mathbf{I} - S_M)\hat{k}_t + (\mathbf{I} - \alpha)(\mathbf{I} - S_M)\hat{l}_t + S_M\varepsilon_Q(\mathbf{I} - S_1^M)\hat{p}_t\Bigg].
\end{aligned}$$

Collecting terms, we have that:

$$\begin{aligned}
0 &= \delta_C^{-1}\tilde{S}_C^Q\hat{c}_{t+1} + (\mathbf{I} - \delta_C^{-1})\tilde{S}_C^Q\hat{c}_t \\
&+ \left[(\varepsilon_Q - 1)\tilde{S}_M^Q T_2 - \left(\mathbf{I} - \tilde{S}_M^Q T_2\right)\left[\Sigma^{-1} - S_M\left[\varepsilon_Q(\Sigma - \mathbf{I}) + \mathbf{I}\right]\Sigma^{-1}\right]^{-1}(\mathbf{I} + S_M(\varepsilon_Q - 1))\right]\hat{a}_t \\
&+ \tilde{S}_X^Q T_2\delta_K^{-1}\hat{k}_{t+1} + \left[\tilde{S}_X^Q T_2(1 - \delta_K^{-1}) - \left(\mathbf{I} - \tilde{S}_M^Q T_2\right)\left[\Sigma^{-1} - S_M\left[\varepsilon_Q(\Sigma - \mathbf{I}) + \mathbf{I}\right]\Sigma^{-1}\right]^{-1}\alpha(\mathbf{I} - S_M)\right]\hat{k}_t \\
&- \left\{\left(\mathbf{I} - \tilde{S}_M^Q T_2\right)\left[\Sigma^{-1} - S_M\left[\varepsilon_Q(\Sigma - \mathbf{I}) + \mathbf{I}\right]\Sigma^{-1}\right]^{-1}(\mathbf{I} - \alpha)(\mathbf{I} - S_M)\right\}\hat{l}_t \\
&+ \Bigg[\varepsilon_Q\tilde{S}_M^Q T_2(\mathbf{I} - S_1^M) + \varepsilon_M\tilde{S}_M^Q[T_2S_1^M - T_1] + \varepsilon_X\tilde{S}_X^Q[T_2S_1^X - T_1] \\
&- \left(\mathbf{I} - \tilde{S}_M^Q T_2\right)\left[\Sigma^{-1} - S_M\left[\varepsilon_Q(\Sigma - \mathbf{I}) + \mathbf{I}\right]\Sigma^{-1}\right]^{-1}S_M\varepsilon_Q(\mathbf{I} - S_1^M)\Bigg]\hat{p}_t.
\end{aligned}$$

We also have the following equation, which is unaffected by the degree of returns to scale:

$$\hat{p}_t = \beta(\mathbf{I} - \delta_C)\hat{p}_{t+1} - \frac{1}{\varepsilon_D}(\mathbf{I} - \beta(\mathbf{I} - \delta_C))\left[\mathbf{I} + S_I^C(\varepsilon_D - 1)\right]\hat{c}_{t+1}.$$

We also have the following equation:

$$S_1^X \hat{p}_t = \left[\tilde{\beta}\mathbf{I} + \beta(1-\delta_K)S_1^X\right]\hat{p}_{t+1} + \frac{\tilde{\beta}}{\varepsilon_Q}\left[\varepsilon_Q(\Sigma - \mathbf{I}) + \mathbf{I}\right]\Sigma^{-1}\hat{q}_{t+1}$$
$$+ \tilde{\beta}\frac{\varepsilon_Q - 1}{\varepsilon_Q}\hat{a}_{t+1} + \tilde{\beta}\left(-\mathbf{I} + \alpha\frac{\varepsilon_Q - 1}{\varepsilon_Q}\right)\hat{k}_{t+1} + \tilde{\beta}(\mathbf{I} - \alpha)\frac{\varepsilon_Q - 1}{\varepsilon_Q}\hat{l}_{t+1}.$$

Plugging in for $\hat{q}_t$ from equation (C.11), we have that:

$$\begin{aligned} S_1^X \hat{p}_t = & \left[\tilde{\beta}\mathbf{I} + \beta(1-\delta_K)S_1^X + \mho S_M \varepsilon_Q(\mathbf{I} - S_1^M)\right]\hat{p}_{t+1} \\ & + \left[\tilde{\beta}\frac{\varepsilon_Q - 1}{\varepsilon_Q} + \mho(\mathbf{I} + S_M(\varepsilon_Q - 1))\right]\hat{a}_{t+1} \\ & + \left[\tilde{\beta}\left(-\mathbf{I} + \alpha\frac{\varepsilon_Q - 1}{\varepsilon_Q}\right) + \mho\alpha(\mathbf{I} - S_M)\right]\hat{k}_{t+1} \\ & + \left[\tilde{\beta}(\mathbf{I} - \alpha)\frac{\varepsilon_Q - 1}{\varepsilon_Q} + \mho(\mathbf{I} - \alpha)(\mathbf{I} - S_M)\right]\hat{l}_{t+1} \end{aligned}$$

where $\mho \equiv \frac{\tilde{\beta}}{\varepsilon_Q}\left[\varepsilon_Q(\Sigma - \mathbf{I}) + \mathbf{I}\right]\Sigma^{-1}\left[\Sigma^{-1} - S_M\left[\varepsilon_Q(\Sigma - \mathbf{I}) + \mathbf{I}\right]\Sigma^{-1}\right]^{-1}$.

Finally, in the equation involving $\hat{l}_t$, we have that:

$$\frac{1}{\varepsilon_{LS}}S^L\hat{l}_t = \hat{p}_t + \frac{\varepsilon_Q - 1}{\varepsilon_Q}\hat{a}_t$$
$$+ \frac{1}{\varepsilon_Q}\left[\varepsilon_Q(\Sigma - \mathbf{I}) + \mathbf{I}\right]\Sigma^{-1}\hat{q}_t + \frac{\varepsilon_Q - 1}{\varepsilon_Q}\alpha\hat{k}_t + \frac{\alpha - \mathbf{I} - \alpha\varepsilon_Q}{\varepsilon_Q}\hat{l}_t,$$

which we can rewrite by substituting in for $\hat{q}_t$ using equation (C.11). Letting $\Theta \equiv \left[\Sigma^{-1} - S_M\left[\varepsilon_Q(\Sigma - \mathbf{I}) + \mathbf{I}\right]\Sigma^{-1}\right]^{-1}$, we have that:

$$\begin{aligned} \hat{q}_t = \Theta \Bigg[& (\mathbf{I} + S_M(\varepsilon_Q - 1))\hat{a}_t + \alpha(\mathbf{I} - S_M)\hat{k}_t + (\mathbf{I} - \alpha)(\mathbf{I} - S_M)\hat{l}_t \\ & + S_M\varepsilon_Q(\mathbf{I} - S_1^M)\hat{p}_t\Bigg], \end{aligned} \tag{C.12}$$

which can be rearranged to yield:

$$\begin{aligned} & \left[\frac{1}{\varepsilon_{LS}}S^L - \frac{\alpha - \mathbf{I} - \alpha\varepsilon_Q}{\varepsilon_Q} - \frac{1}{\varepsilon_Q}\left[\varepsilon_Q(\Sigma - \mathbf{I}) + \mathbf{I}\right]\Sigma^{-1}\Theta(\mathbf{I} - \alpha)(\mathbf{I} - S_M)\right]\hat{l}_t \\ = & \left[\mathbf{I} + \left[\varepsilon_Q(\Sigma - \mathbf{I}) + \mathbf{I}\right]\Sigma^{-1}\Theta S_M\varepsilon_Q(\mathbf{I} - S_1^M)\right]\hat{p}_t \\ & + \left[\frac{\varepsilon_Q - 1}{\varepsilon_Q} + \frac{1}{\varepsilon_Q}\left[\varepsilon_Q(\Sigma - \mathbf{I}) + \mathbf{I}\right]\Sigma^{-1}\Theta(\mathbf{I} + S_M(\varepsilon_Q - 1))\right]\hat{a}_t \\ & + \left[\frac{\varepsilon_Q - 1}{\varepsilon_Q}\alpha + \frac{1}{\varepsilon_Q}\left[\varepsilon_Q(\Sigma - \mathbf{I}) + \mathbf{I}\right]\Sigma^{-1}\Theta\alpha(\mathbf{I} - S_M)\right]\hat{k}_t. \end{aligned}$$

Simplifying the notation, the above equation can be rewritten as:

$$\hat{l}_t = \mathcal{L}_p \hat{p}_t + \mathcal{L}_a \hat{a}_t + \mathcal{L}_k \hat{k}_t. \tag{C.13}$$

We can similarly simplify the notation in the remaining three equations:

$$0 = \mathcal{C}_c^1 \hat{c}_{t+1} + \mathcal{C}_c \hat{c}_t + \mathcal{C}_a \hat{a}_t + \mathcal{C}_{k+} \hat{k}_{t+1} + \mathcal{C}_k \hat{k}_t + \mathcal{C}_p \hat{p}_t + \mathcal{C}_l \hat{l}_t, \tag{C.14}$$

$$\hat{p}_t = \beta(\mathbf{I} - \delta_C)\hat{p}_{t+1} - \frac{1}{\varepsilon_D}(\mathbf{I} - \beta(\mathbf{I} - \delta_C))[\mathbf{I} + S_I^C(\varepsilon_D - 1)]\hat{c}_{t+1}, \tag{C.15}$$

$$S_1^X \hat{p}_t = \mathcal{P}_p^+ \hat{p}_{t+1} + \mathcal{P}_a \hat{a}_{t+1} + \mathcal{P}_k \hat{k}_{t+1} + \mathcal{P}_l \hat{l}_{t+1}. \tag{C.16}$$

Substituting for $\hat{l}_t$ using equation (C.13) into equations (C.14)–(C.16), we have that:

$$\begin{aligned}
0 &= \mathcal{C}_c^1 \hat{c}_{t+1} + \mathcal{C}_c \hat{c}_t + \left[\mathcal{C}_a + \mathcal{C}_l \mathcal{L}_a\right] \hat{a}_t + \mathcal{C}_{k+} \hat{k}_{t+1} + \left[\mathcal{C}_k + \mathcal{C}_l \mathcal{L}_k\right] \hat{k}_t + \left[\mathcal{C}_p + \mathcal{C}_l \mathcal{L}_p\right] \hat{p}_t, \\
\hat{p}_t &= \beta(\mathbf{I} - \delta_C)\hat{p}_{t+1} - \frac{1}{\varepsilon_D}(\mathbf{I} - \beta(\mathbf{I} - \delta_C))[\mathbf{I} + S_I^C(\varepsilon_D - 1)]\hat{c}_{t+1}, \\
S_1^X \hat{p}_t &= \left[\mathcal{P}_p^+ + \mathcal{P}_l \mathcal{L}_p\right] \hat{p}_{t+1} + \left[\mathcal{P}_a + \mathcal{P}_l \mathcal{L}_a\right] \hat{a}_{t+1} + \left[\mathcal{P}_k + \mathcal{P}_l \mathcal{L}_k\right] \hat{k}_{t+1}.
\end{aligned} \tag{C.17}$$

Since we are assuming away durable goods (i.e. $\delta_C = 1$), we have from equation (C.15) that $\hat{c}_{t+1}$ is given by:

$$\hat{c}_{t+1} = -\varepsilon_D[\mathbf{I} + S_I^C(\varepsilon_D - 1)]^{-1}\hat{p}_t. \tag{C.18}$$

Plugging in for $\hat{c}_{t+1}$ above, we have that:

$$\begin{aligned}
0 = {} & \mathcal{C}_c \hat{c}_t + \left[\mathcal{C}_a + \mathcal{C}_l \mathcal{L}_a\right] \hat{a}_t + \mathcal{C}_{k+} \hat{k}_{t+1} + \left[\mathcal{C}_k + \mathcal{C}_l \mathcal{L}_k\right] \hat{k}_t \\
& + \left[-\varepsilon_D \mathcal{C}_c^1 [\mathbf{I} + S_I^C(\varepsilon_D - 1)]^{-1} + \mathcal{C}_p + \mathcal{C}_l \mathcal{L}_p\right] \hat{p}_t.
\end{aligned}$$

The log-linearised equation for consumption as a function of prices and demand shocks is:[19]

$$\hat{c}_t = \hat{d}_t - \varepsilon_D \left[\mathbf{I} + S_I^C(\varepsilon_D - 1)\right]^{-1} \hat{p}_t. \tag{C.19}$$

By substituting out $\hat{c}_t$ using the above equation, we obtain:

$$\begin{aligned}
0 = {} & \mathcal{C}_c \hat{d}_t + \left[\mathcal{C}_a + \mathcal{C}_l \mathcal{L}_a\right] \hat{a}_t + \mathcal{C}_{k+} \hat{k}_{t+1} + \left[\mathcal{C}_k + \mathcal{C}_l \mathcal{L}_k\right] \hat{k}_t \\
& + \left[-\varepsilon_D \left(\mathcal{C}_c^1 + \mathcal{C}_c\right) [\mathbf{I} + S_I^C(\varepsilon_D - 1)]^{-1} + \mathcal{C}_p + \mathcal{C}_l \mathcal{L}_p\right] \hat{p}_t.
\end{aligned} \tag{C.20}$$

[19]For a derivation of this equation, see page 72 in Appendix F in Atalay (2017).

Note that equations (C.17) and (C.20) can be written in matrix form as:

$$\begin{bmatrix} \hat{p}_{t+1} \\ \hat{k}_{t+1} \end{bmatrix} = \begin{bmatrix} \mathbf{\Psi}_{pp} & \mathbf{\Psi}_{pk} \\ \mathbf{\Psi}_{kp} & \mathbf{\Psi}_{kk} \end{bmatrix} \begin{bmatrix} \hat{p}_t \\ \hat{k}_t \end{bmatrix} + \begin{bmatrix} \mathbf{\Phi}_{pa} & \mathbf{\Psi}_{pd} \\ \mathbf{\Phi}_{ka} & \mathbf{\Psi}_{kd} \end{bmatrix} \begin{bmatrix} \hat{a}_t \\ \hat{d}_t \end{bmatrix},$$

where the multiplying matrices (in bold) are defined by solving equations (C.17) and (C.20) for $\hat{p}_{t+1}$ and $\hat{k}_{t+1}$, respectively, in terms of $\hat{k}_t$, $\hat{p}_t$, and the shocks ($\hat{a}_t$ and $\hat{d}_t$). Using the Blanchard–Kahn decomposition, one can obtain the following two equations (see Appendix F.4 in Atalay (2017)):

$$\hat{k}_{t+1} = \mathbf{M}_{kk}\hat{k}_t + [\mathbf{M}_{ka}, \mathbf{M}_{kd}] \begin{bmatrix} \hat{a}_t \\ \hat{d}_t \end{bmatrix}, \tag{C.21}$$

$$\hat{p}_t = \mathbf{\Psi}_{21}^{-1}\hat{k}_{t+1} - \mathbf{\Psi}_{21}^{-1}\mathbf{\Psi}_{22}\hat{k}_t - \mathbf{\Psi}_{21}^{-1}\mathbf{\Phi}_2^d \begin{bmatrix} \hat{a}_t \\ \hat{d}_t \end{bmatrix}. \tag{C.22}$$

By substituting for $\hat{k}_{t+1}$ in equation (C.22) using equation (C.21), we can express $\hat{p}_t$ as a function of the state variable, $\hat{k}_t$, and the shocks, i.e.:

$$\hat{p}_t = \mathbf{P}_k\hat{k}_t + \mathbf{P}_a\hat{a}_t + \mathbf{P}_d\hat{d}_t. \tag{C.23}$$

Next, by substituting for $\hat{l}_t$ in equation (C.12) using equation (C.13), we obtain $\hat{q}_t$ as a function of $\hat{k}_t$, $\hat{p}_t$, and the shocks. We can then in turn plug in for $\hat{p}_t$ (using equation (C.23) above) and express $\hat{q}_t$ also solely as a function of the state variable, $\hat{k}_t$, and the shocks:

$$\hat{q}_t = \mathbf{\Phi}_k\hat{k}_t + \mathbf{\Phi}_a\hat{a}_t + \mathbf{\Phi}_d\hat{d}_t, \tag{C.24}$$

where the matrices in bold depend (non-linearly) on model parameters only. Taking differences on both sides of equation (C.24) and denoting $\Delta\hat{a}_t$ by ω_t^A and $\Delta\hat{d}_t$ by ω_t^D, we can derive the equation for $\Delta\hat{q}_t$ in the top row of equation (13) in the main text:

$$\Delta\hat{q}_t = \mathbf{\Phi}_k\Delta\hat{k}_t + \mathbf{\Phi}_a\omega_t^A + \mathbf{\Phi}_d\omega_t^D. \tag{C.25}$$

C.3 Deriving the log-linearised equation for value-added

By definition, nominal value-added in industry i in period t is given by:

$$VA_{it} = P_{it}Q_{it} - M_{it}P_{it}^M. \tag{C.26}$$

Recall that the intermediate input bundle, M_{it}, is given by equation (3) in the main text. Recall also the first-order condition with respect to M_{ij} derived in Section C.1. Adding time subscripts to that equation and re-introducing shocks (which are normalised to unity in steady state), we have that:

$$M_{ijt}^{\frac{\varepsilon_M-1}{\varepsilon_M}} = \left[\left(\frac{P_{it}}{P_{jt}}\right)(A_{it})^{\frac{\varepsilon_Q-1}{\varepsilon_Q}}\left(\frac{\mu_i}{M_{it}}\right)^{\frac{1}{\varepsilon_Q}}\left(M_{it}\Gamma_{ij}^{M}\right)^{\frac{1}{\varepsilon_M}}\eta_i Q_{it}^{\frac{\varepsilon_Q(\eta_i-1)+1}{\varepsilon_Q\eta_i}}\right]^{\varepsilon_M-1}.$$

Pre-multiplying both sides by $(\Gamma_{ij}^{M})^{\frac{1}{\varepsilon_M}}$, summing over all suppliers $j \in \{1,\ldots,N\}$, and raising the resulting sums on both sides to the power of $\varepsilon_M/(\varepsilon_M-1)$, one obtains the following equation:

$$M_{it} = P_{it}^{\varepsilon_M}(A_{it})^{\frac{(\varepsilon_Q-1)\varepsilon_M}{\varepsilon_Q}}\left(\frac{\mu_i}{M_{it}}\right)^{\frac{\varepsilon_M}{\varepsilon_Q}} M_{it}\left[\eta_i Q_{it}^{\frac{\varepsilon_Q(\eta_i-1)+1}{\varepsilon_Q\eta_i}}\right]^{\varepsilon_M}(P_{it}^{M})^{-\varepsilon_M},$$

which can be rewritten as:

$$M_{it} = \left(\frac{P_{it}}{P_{it}^{M}}\right)^{\varepsilon_Q}(A_{it})^{\varepsilon_Q-1}\mu_i\eta_i^{\varepsilon_Q}Q_{it}^{\frac{\varepsilon_Q(\eta_i-1)+1}{\eta_i}}. \tag{C.27}$$

Using the above equation, we can substitute in for M_{it} in equation (C.26), which gives:

$$VA_{it} = P_{it}Q_{it}\left[1-\left(\frac{P_{it}^{M}}{P_{it}}\right)^{1-\varepsilon_Q}(A_{it})^{\varepsilon_Q-1}\mu_i\eta_i^{\varepsilon_Q}Q_{it}^{\frac{\varepsilon_Q(\eta_i-1)+1-\eta_i}{\eta_i}}\right].$$

Therefore, real value-added of industry i in period t (V_{it}) is given by:

$$V_{it} \equiv \frac{VA_{it}}{P_{it}} = Q_{it}\left[1-\left(\frac{P_{it}^{M}}{P_{it}}\right)^{1-\varepsilon_Q}(A_{it})^{\varepsilon_Q-1}\mu_i\eta_i^{\varepsilon_Q}Q_{it}^{\frac{\varepsilon_Q(\eta_i-1)+1-\eta_i}{\eta_i}}\right]. \tag{C.28}$$

In order to log-linearise equation (C.28), we can rewrite it as:

$$\frac{V_{it}}{Q_{it}} = 1-\left(\frac{P_{it}^{M}}{P_{it}}\right)^{1-\varepsilon_Q}(A_{it})^{\varepsilon_Q-1}\mu_i\eta_i^{\varepsilon_Q}Q_{it}^{\frac{\varepsilon_Q(\eta_i-1)+1-\eta_i}{\eta_i}},$$

which can in turn be rewritten as:

$$\frac{V_i}{Q_i}e^{\hat{v}_{it}-\hat{q}_{it}} = 1-\left(\frac{P_i^{M}}{P_i}\right)^{1-\varepsilon_Q}e^{(1-\varepsilon_Q)(\hat{p}_{it}^{M}-\hat{p}_{it})}e^{(\varepsilon_Q-1)\hat{a}_{it}}\mu_i\eta_i^{\varepsilon_Q}Q_i^{\frac{\varepsilon_Q(\eta_i-1)+1-\eta_i}{\eta_i}}e^{\left(\frac{\varepsilon_Q(\eta_i-1)+1-\eta_i}{\eta_i}\right)\hat{q}_{it}}.$$

To proceed, we make use of the standard calibration target adopted by Atalay (2017), $V_i/Q_i = 1-\mu_i$,[20] together with the approximation that $e^x \approx 1+x$ when x is small. We

[20]Under decreasing returns to scale, this calibration imposes a non-trivial steady-state restriction; see equation (C.29) below. The interpretation is that μ_i is defined as the data target $1-V_i/Q_i$ at the long-run average; the production-function parameter is then identified jointly with the rest of the steady-state system. In the constant-returns-to-scale limit ($\eta_i = 1$), this collapses to the familiar identification of μ_i

thus get:

$$(1-\mu_i)(1+\hat{v}_{it}-\hat{q}_{it}) = 1 - \left(\frac{P_i^M}{P_i}\right)^{1-\varepsilon_Q} \left[1+(1-\varepsilon_Q)\left(\hat{p}_{it}^M - \hat{p}_{it}\right)\right]$$
$$\cdot\, \mu_i \eta_i^{\varepsilon_Q} Q_i^{\frac{\varepsilon_Q(\eta_i-1)+1-\eta_i}{\eta_i}} \left(1+(\varepsilon_Q-1)\hat{a}_{it}\right)\left[1+\left(\frac{\varepsilon_Q(\eta_i-1)+1-\eta_i}{\eta_i}\right)\hat{q}_{it}\right].$$

Next, using the approximation that $\hat{x}_t\hat{y}_t \approx 0$ for any $\hat{x}_t$ and $\hat{y}_t$ denoting (log) deviations of x_t and y_t from their steady-state values, we obtain:

$$(1-\mu_i)(1+\hat{v}_{it}-\hat{q}_{it}) = 1 - \left(\frac{P_i^M}{P_i}\right)^{1-\varepsilon_Q} \mu_i \eta_i^{\varepsilon_Q} Q_i^{\frac{\varepsilon_Q(\eta_i-1)+1-\eta_i}{\eta_i}}$$
$$\cdot \left[1+(1-\varepsilon_Q)\left(\hat{p}_{it}^M - \hat{p}_{it}\right) + (\varepsilon_Q-1)\hat{a}_{it} + \left(\frac{\varepsilon_Q(\eta_i-1)+1-\eta_i}{\eta_i}\right)\hat{q}_{it}\right].$$

Note that in steady state—in which all of the variables with a hat in the above equation equal zero—we have that:

$$1-\mu_i \;=\; 1-\left(\frac{P_i^M}{P_i}\right)^{1-\varepsilon_Q} \mu_i\, \eta_i^{\varepsilon_Q}\, Q_i^{\frac{\varepsilon_Q(\eta_i-1)+1-\eta_i}{\eta_i}}. \tag{C.29}$$

Equation (C.29) is the steady-state restriction that the calibration imposes: given μ_i, η_i, and ε_Q, it pins down the steady-state ratio P_i^M/P_i jointly with the rest of the steady-state system in (C.4)–(C.10). In the constant-returns-to-scale limit ($\eta_i = 1$), the exponent on Q_i is zero and the equation reduces to $(P_i^M/P_i)^{1-\varepsilon_Q} = 1$, i.e. $P_i^M = P_i$ in steady state (provided $\varepsilon_Q \neq 1$), recovering the calibration in Atalay (2017). In the special case $\varepsilon_Q = 1$ combined with non-trivial DRS, equation (C.29) collapses to $\eta_i = 1$, so the calibration is non-degenerate only when $\varepsilon_Q \neq 1$—a feature that is consistent with our preferred (and robustness) calibrations of ε_Q. Therefore, cancelling out the steady-state terms, we get:

$$(1-\mu_i)(\hat{v}_{it}-\hat{q}_{it}) = -\left(\frac{P_i^M}{P_i}\right)^{1-\varepsilon_Q} \mu_i \eta_i^{\varepsilon_Q} Q_i^{\frac{\varepsilon_Q(\eta_i-1)+1-\eta_i}{\eta_i}}$$
$$\cdot \left[(1-\varepsilon_Q)\left(\hat{p}_{it}^M - \hat{p}_{it}\right) + (\varepsilon_Q-1)\hat{a}_{it} + \left(\frac{\varepsilon_Q(\eta_i-1)+1-\eta_i}{\eta_i}\right)\hat{q}_{it}\right].$$

Taking value-added on the left-hand side, we finally obtain:

$$\hat{v}_{it} = \hat{q}_{it} - \left(\frac{P_i^M}{P_i}\right)^{1-\varepsilon_Q} \frac{\mu_i}{1-\mu_i} \eta_i^{\varepsilon_Q} Q_i^{\frac{\varepsilon_Q(\eta_i-1)+1-\eta_i}{\eta_i}}$$
$$\cdot \left[(1-\varepsilon_Q)\left(\hat{p}_{it}^M - \hat{p}_{it}\right) + (\varepsilon_Q-1)\hat{a}_{it} + \left(\frac{\varepsilon_Q(\eta_i-1)+1-\eta_i}{\eta_i}\right)\hat{q}_{it}\right]. \tag{C.30}$$

with the empirical intermediate revenue share.

Note that in the presence of constant returns to scale ($\eta_i = 1$), equation (C.30) becomes:

$$\hat{v}_{it} = \hat{q}_{it} - \frac{\mu_i}{1-\mu_i}\left[(1-\varepsilon_Q)\left(\hat{p}^M_{it} - \hat{p}_{it}\right) + (\varepsilon_Q - 1)\hat{a}_{it}\right].$$

In matrix form, we can write equation (C.30) as:

$$\hat{v}_t = \mathcal{V}_q \hat{q}_t + \mathcal{V}_p \hat{p}_t + \mathcal{V}_a \hat{a}_t, \tag{C.31}$$

where we use the relationship between the vector of output prices and the prices of intermediate input bundles in each industry.

Recall from equations (C.23) and (C.24) that both $\hat{q}_t$ and $\hat{p}_t$ can be expressed as functions of $\hat{k}_t$ and the shocks only. Therefore, we can substitute for $\hat{q}_t$ and $\hat{p}_t$ in equation (C.31) to express value-added as a function of $\hat{k}_t$ and the shocks. In other words, one obtains:

$$\hat{v}_t = \mathbf{V}_k \hat{k}_t + \mathbf{V}_a \hat{a}_t + \mathbf{V}_d \hat{d}_t. \tag{C.32}$$

As long as $\mathbf{V}_k^{-1}$ exists, we can equivalently write the above equation as:

$$\hat{k}_t = \mathbf{V}_k^{-1}\hat{v}_t - \mathbf{V}_k^{-1}\mathbf{V}_a \hat{a}_t - \mathbf{V}_k^{-1}\mathbf{V}_d \hat{d}_t. \tag{C.33}$$

We thus have that one period ahead:

$$\begin{aligned}
\hat{v}_{t+1} &= \mathbf{V}_k \hat{k}_{t+1} + \mathbf{V}_a \hat{a}_{t+1} + \mathbf{V}_d \hat{d}_{t+1} \\
&= \mathbf{V}_k \left(\mathbf{M}_{kk}\hat{k}_t + \mathbf{M}_{ka}\hat{a}_t + \mathbf{M}_{kd}\hat{d}_t\right) + \mathbf{V}_a \hat{a}_{t+1} + \mathbf{V}_d \hat{d}_{t+1},
\end{aligned}$$

where we make use of equation (C.21) in the second line. By plugging in for $\hat{k}_t$ in the above equation using equation (C.33), we therefore obtain $\hat{v}_{t+1}$ as a function of the shocks at time $t+1$ ($\hat{a}_{t+1}$ and $\hat{d}_{t+1}$), the shocks at time t ($\hat{a}_t$ and $\hat{d}_t$), and $\hat{v}_t$. Taking differences on both sides of this resulting equation, we obtain the top row of the VARMA(1,1) representation in Section 3.4 of the main text:

$$\Delta\hat{v}_{t+1} = \tilde{\mathbf{V}}_v \Delta\hat{v}_t + [\mathbf{V}_a \;\; \mathbf{V}_d]\begin{bmatrix}\omega^A_{t+1} \\ \omega^D_{t+1}\end{bmatrix} + [\tilde{\mathbf{V}}_a \;\; \tilde{\mathbf{V}}_d]\begin{bmatrix}\omega^A_t \\ \omega^D_t\end{bmatrix}. \tag{C.34}$$

Starting from equation (C.23) for the vector of prices, $\hat{p}_t$, and following the analogous steps shown above for the vector of value-added (in equation (C.32)), we obtain the bottom row of the same VARMA(1,1) representation.

C.4 Deriving the log-linear equation for outdegrees

The first-order weighted outdegree of industry j at time t is defined by:

$$D_{jt}^{\text{out}} = \sum_{i=1}^{N} \omega_{ijt}.$$

Log-linearising the above summation, we have that:

$$\hat{d}_{jt}^{\text{out}} = \sum_{i=1}^{N} \frac{\omega_{ij}}{D_j^{\text{out}}} \hat{\omega}_{ijt},$$

where

$$\hat{\omega}_{ijt} = \hat{m}_{ijt} + \hat{p}_{jt} - \hat{m}_{it} - \hat{p}_{it}^{M}.$$

We can thus show that

$$\hat{d}_t^{\text{out}} = \mathbf{H}\left[\hat{m}_t - T_2\hat{\mathbb{M}}_t + T_1\hat{p}_t - T_2\hat{p}_t^{M}\right],$$

where $\mathbf{H}_{N\times N^2}$ is the selector matrix that maps the stacked input shares into outdegrees. Specifically, with $\hat{m}_t$ stacked consumer-first (consumer index running slow, supplier index running fast), the j-th row of $\mathbf{H}$ has $\omega_{ij}/D_j^{\text{out}}$ at positions $(i-1)N+j$ for $i = 1, \ldots, N$ and zeros elsewhere (steady-state values of input shares and outdegrees). Note that above:

$$\hat{m}_t = \begin{bmatrix} \hat{m}_{11t} \\ \hat{m}_{12t} \\ \cdots \\ \hat{m}_{1Nt} \\ \cdots \\ \hat{m}_{N1t} \\ \hat{m}_{N2t} \\ \cdots \\ \hat{m}_{NNt} \end{bmatrix}_{N^2\times 1}, \quad \hat{\mathbb{M}}_t = \begin{bmatrix} \hat{m}_{1t} \\ \hat{m}_{2t} \\ \cdots \\ \hat{m}_{Nt} \end{bmatrix}_{N\times 1}, \quad \hat{p}_t = \begin{bmatrix} \hat{p}_{1t} \\ \hat{p}_{2t} \\ \cdots \\ \hat{p}_{Nt} \end{bmatrix}_{N\times 1}, \quad \hat{p}_t^{M} = \begin{bmatrix} \hat{p}_{1t}^{M} \\ \hat{p}_{2t}^{M} \\ \cdots \\ \hat{p}_{Nt}^{M} \end{bmatrix}_{N\times 1},$$

where the multiplying matrices are given by $T_1 = \mathbf{1}_N \otimes \mathbf{I}_N$ and $T_2 = \mathbf{I}_N \otimes \mathbf{1}_N$, where $\mathbf{1}_N$ is the $N \times 1$ vector of ones, $\mathbf{I}_N$ is the $N \times N$ identity matrix, and $\otimes$ denotes the Kronecker product.[21]

[21] Under the consumer-first stacking adopted here (consumer index i runs slow, supplier index j runs fast), $T_1 = \mathbf{1}_N \otimes \mathbf{I}_N$ takes an N-vector indexed by suppliers and broadcasts it across the consumer dimension, so that entry $(i-1)N+j$ of $T_1\hat{p}_t$ equals $\hat{p}_{jt}$. Conversely, $T_2 = \mathbf{I}_N \otimes \mathbf{1}_N$ takes an N-vector indexed by consumers and broadcasts it across the supplier dimension, so that entry $(i-1)N+j$ of $T_2\hat{a}_t$

Under the assumption of constant returns to scale, the log-linear equation for the vector of intermediate input bundles in each industry is given by:[22]

$$\hat{m}_t = \frac{\varepsilon_M}{\varepsilon_Q}(\varepsilon_Q - 1)T_2\hat{a}_t + \frac{\varepsilon_M}{\varepsilon_Q}T_2\hat{q}_t + \left(1 - \frac{\varepsilon_M}{\varepsilon_Q}\right)T_2\hat{\mathbb{M}}_t + \varepsilon_M[T_2 - T_1]\hat{p}_t.$$

Instead, allowing for potentially non-constant returns to scale, we will have that:

$$\hat{m}_t = \frac{\varepsilon_M}{\varepsilon_Q}(\varepsilon_Q - 1)T_2\hat{a}_t + \frac{\varepsilon_M}{\varepsilon_Q}T_2\left[\varepsilon_Q(\Sigma - \mathbf{I}) + \mathbf{I}\right]\Sigma^{-1}\hat{q}_t + \left(1 - \frac{\varepsilon_M}{\varepsilon_Q}\right)T_2\hat{\mathbb{M}}_t + \varepsilon_M[T_2 - T_1]\hat{p}_t. \tag{C.35}$$

Recall from above that the $N \times 1$ vector of intermediate input bundles ($\hat{\mathbb{M}}_t$) is given by:

$$\hat{\mathbb{M}}_t = (\varepsilon_Q - 1)\hat{a}_t + \left[\varepsilon_Q(\Sigma - \mathbf{I}) + \mathbf{I}\right]\Sigma^{-1}\hat{q}_t + \varepsilon_Q(\mathbf{I} - S_1^M)\hat{p}_t.$$

Substituting in for $\hat{\mathbb{M}}_t$ in equation (C.35) and simplifying, we have that:

$$\hat{m}_t = (\varepsilon_Q - 1)T_2\hat{a}_t + T_2\left[\varepsilon_Q(\Sigma - \mathbf{I}) + \mathbf{I}\right]\Sigma^{-1}\hat{q}_t + \left[\left(1 - \frac{\varepsilon_M}{\varepsilon_Q}\right)\varepsilon_Q T_2(\mathbf{I} - S_1^M) + \varepsilon_M[T_2 - T_1]\right]\hat{p}_t.$$

By definition, we have that:

$$\hat{d}_t^{\text{out}} = \mathbf{H}\left[\hat{m}_t - T_2\hat{\mathbb{M}}_t + T_1\hat{p}_t - T_2\hat{p}_t^M\right]_{N^2\times 1}. \tag{C.36}$$

Substituting in for $\hat{m}_t$, $\hat{\mathbb{M}}_t$, and $\hat{p}_t^M$ (which, by definition, equals $S_1^M\hat{p}_t$), we have that:

$$\begin{aligned}\hat{d}_t^{\text{out}} = \mathbf{H}&\left[(\varepsilon_Q - 1)T_2\hat{a}_t + T_2\left[\varepsilon_Q(\Sigma - \mathbf{I}) + \mathbf{I}\right]\Sigma^{-1}\hat{q}_t + \left(\left(1 - \frac{\varepsilon_M}{\varepsilon_Q}\right)\varepsilon_Q T_2(\mathbf{I} - S_1^M) + \varepsilon_M[T_2 - T_1]\right)\hat{p}_t\right] \\ &- \mathbf{H}\left[(\varepsilon_Q - 1)T_2\hat{a}_t + T_2\left[\varepsilon_Q(\Sigma - \mathbf{I}) + \mathbf{I}\right]\Sigma^{-1}\hat{q}_t + \varepsilon_Q T_2(\mathbf{I} - S_1^M)\hat{p}_t\right] + \mathbf{H}\left(T_1 - T_2S_1^M\right)\hat{p}_t,\end{aligned}$$

which can be further simplified to yield:

$$\begin{aligned}\hat{d}_t^{\text{out}} &= \mathbf{H}\left[\left(1 - \frac{\varepsilon_M}{\varepsilon_Q}\right)\varepsilon_Q T_2(\mathbf{I} - S_1^M) + \varepsilon_M(T_2 - T_1) - \varepsilon_Q T_2(\mathbf{I} - S_1^M) + \left(T_1 - T_2S_1^M\right)\right]\hat{p}_t \\ &\equiv \mathbf{D}\hat{p}_t. \end{aligned} \tag{C.37}$$

Therefore, as equation (C.37) shows, the relationship between outdegrees and prices is independent of the returns to scale. This was also the case in the simplified model in Section 3.2.

Finally, recall from equation (C.23) that we can express the vector of prices, $\hat{p}_t$,

equals $\hat{a}_{it}$.

[22] For a derivation of this equation, see page 57 in Atalay (2017).

as a function of the state variable, $\hat{k}_t$, and the shocks. By substituting for $\hat{p}_t$ into equation (C.37), we can obtain an equation for the outdegrees as a function of the state variable and shocks only, shown as equation (14) in the main text:

$$\begin{aligned} \hat{d}_t^{\text{out}} &= \mathbf{D}\left[\mathbf{P}_k\hat{k}_t + \mathbf{P}_a\hat{a}_t + \mathbf{P}_d\hat{d}_t\right], \\ \Delta\hat{d}_t^{\text{out}} &= \mathbf{D}_k\Delta\hat{k}_t + \mathbf{D}_a\omega_t^A + \mathbf{D}_d\omega_t^D. \end{aligned} \tag{C.38}$$

C.5 Deriving the log-linearised equation for Domar weights

We follow Atalay (2017) in normalising steady-state aggregate labour to $L = \sum_{i=1}^N L_i = 1$ (so that, equivalently, $L^{1/\varepsilon_{LS}} = 1$).[23] Under constant returns to scale, profits vanish and this normalisation follows directly from the household's labour-supply FOC; under decreasing returns to scale, it is an explicit calibration choice that, combined with the FOC $L^{1/\varepsilon_{LS}} = W/Y$ (where $Y = WL + r^K K + \sum_{i=1}^N \Pi_i$ is total household income), pins down the steady-state ratio W/Y to unity. Because steady-state labour rather than the aggregate price index is the numeraire, the latter cannot also be normalised. By definition, nominal GDP is given by:

$$NGDP_t = \sum_{j=1}^N P_{jt}V_{jt},$$

where V_{jt} is the value-added of industry j and P_{jt} the corresponding output price of industry j. In log-linear terms, this is equivalent to:

$$\widehat{NGDP}_t = \sum_{j=1}^N \frac{P_jV_j}{\sum_{j'=1}^N P_{j'}V_{j'}}\left(\hat{p}_{jt} + \hat{v}_{jt}\right).$$

We can write this as

$$\widehat{NGDP}_t = \mathbf{N}\left(\hat{p}_t + \hat{v}_t\right),$$

where $\mathbf{N}$ is a $1 \times N$ row vector that has $\frac{P_jV_j}{\sum_{j'=1}^N P_{j'}V_{j'}}$ in the j-th column.

Now, recall that the Domar weights are given by $\lambda_{jt} \equiv P_{jt}Q_{jt}/NGDP_t$. The log-linear approximation to the deviations of Domar weights from their steady-state values is given by:

$$\hat{\lambda}_t = \hat{p}_t + \hat{q}_t - \widehat{NGDP}_t \cdot \iota, \tag{C.39}$$

[23] Note that the simplified model in Appendix B.1 normalised the consumer price index instead; for the dynamic baseline model we normalise labour, following Atalay (2017).

where $\widehat{NGDP}_t$ denotes the log-deviation of nominal GDP from its steady state, and ι denotes the $N \times 1$ vector of ones.

Therefore, we have that $\hat{\lambda}_t = \hat{p}_t + \hat{q}_t - \bar{\mathbf{N}}\left(\hat{p}_t + \hat{v}_t\right)$, where $\bar{\mathbf{N}} \equiv \iota\,\mathbf{N}$ is the $N \times N$ matrix obtained by stacking the row vector $\mathbf{N}$ vertically N times. This can be written as:

$$\hat{\lambda}_t = \left(\mathbf{I} - \bar{\mathbf{N}}\right)\hat{p}_t + \hat{q}_t - \bar{\mathbf{N}}\hat{v}_t.$$

Using equation (C.31), we can express the Domar weights as:

$$\hat{\lambda}_t = \underbrace{\left(\mathbf{I} - \bar{\mathbf{N}} - \bar{\mathbf{N}}\mathcal{V}_p\right)}_{\mathbf{\Omega}_p}\hat{p}_t + \underbrace{\left(\mathbf{I} - \bar{\mathbf{N}}\mathcal{V}_q\right)}_{\mathbf{\Omega}_q}\hat{q}_t - \underbrace{\bar{\mathbf{N}}\mathcal{V}_a}_{\mathbf{\Omega}_a}\hat{a}_t.$$

We want to express the Domar weights in terms of the state variable (capital) and shocks only. We have that:

$$\begin{aligned}
\hat{\lambda}_t &= \mathbf{\Omega}_p\hat{p}_t + \mathbf{\Omega}_q\hat{q}_t - \mathbf{\Omega}_a\hat{a}_t \\
&= \mathbf{\Omega}_p\left[\mathbf{P}_k\hat{k}_t + \mathbf{P}_a\hat{a}_t + \mathbf{P}_d\hat{d}_t\right] + \mathbf{\Omega}_q\left[\mathbf{\Phi}_k\hat{k}_t + \mathbf{\Phi}_a\hat{a}_t + \mathbf{\Phi}_d\hat{d}_t\right] - \mathbf{\Omega}_a\hat{a}_t \\
&= \mathbf{\Lambda}_k\hat{k}_t + \mathbf{\Lambda}_a\hat{a}_t + \mathbf{\Lambda}_d\hat{d}_t.
\end{aligned}$$

Taking differences on both sides, we obtain the bottom row of equation (13) in Section 3.3:

$$\Delta\hat{\lambda}_t = \mathbf{\Lambda}_k\Delta\hat{k}_t + \mathbf{\Lambda}_a\omega_t^A + \mathbf{\Lambda}_d\omega_t^D. \tag{C.40}$$

C.6 Deriving the log-linearised equation for labour

From the bottom of page 60 in Atalay (2017), we have that:

$$\hat{l}_t = \mathbf{\Phi}_k^L\hat{k}_t + \mathbf{\Phi}_a^L\hat{a}_t + \mathbf{\Phi}_p^L\hat{p}_t.$$

Substituting in for $\hat{p}_t$ using equation (C.23), we have that:

$$\begin{aligned}
\hat{l}_t &= \mathbf{\Phi}_k^L\hat{k}_t + \mathbf{\Phi}_a^L\hat{a}_t + \mathbf{\Phi}_p^L\left[\mathbf{P}_k\hat{k}_t + \mathbf{P}_a\hat{a}_t + \mathbf{P}_d\hat{d}_t\right] \\
&= \mathbf{L}_k\hat{k}_t + \mathbf{L}_a\hat{a}_t + \mathbf{L}_d\hat{d}_t.
\end{aligned} \tag{C.41}$$

One period ahead:

$$\begin{aligned}
\hat{l}_{t+1} &= \mathbf{L}_k\hat{k}_{t+1} + \mathbf{L}_a\hat{a}_{t+1} + \mathbf{L}_d\hat{d}_{t+1} \\
&= \mathbf{L}_k\left(\mathbf{M}_{kk}\hat{k}_t + \mathbf{M}_{ka}\hat{a}_t + \mathbf{M}_{kd}\hat{d}_t\right) + \mathbf{L}_a\hat{a}_{t+1} + \mathbf{L}_d\hat{d}_{t+1},
\end{aligned}$$

where we make use of equation (C.21) in the second line above. As long as $\mathbf{L}_k$ is invertible, we can solve for $\hat{k}_t$ using equation (C.41) and substitute $\hat{k}_t$ from the above equation to obtain:

$$\hat{l}_{t+1} = \mathbf{L}_k \left(\mathbf{M}_{kk} \left[\mathbf{L}_k^{-1}\hat{l}_t - \mathbf{L}_k^{-1}\mathbf{L}_a\hat{a}_t - \mathbf{L}_k^{-1}\mathbf{L}_d\hat{d}_t \right] + \mathbf{M}_{ka}\hat{a}_t + \mathbf{M}_{kd}\hat{d}_t \right) + \mathbf{L}_a\hat{a}_{t+1} + \mathbf{L}_d\hat{d}_{t+1},$$

so we finally get:

$$\Delta\hat{l}_{t+1} = \tilde{\mathbf{L}}_l\Delta\hat{l}_t + \mathbf{L}_a\omega_{t+1}^A + \mathbf{L}_d\omega_{t+1}^D + \tilde{\mathbf{L}}_a\omega_t^A + \tilde{\mathbf{L}}_d\omega_t^D. \tag{C.42}$$

This corresponds to the bottom row of the labour-VARMA filter (18) in Section 5.1 of the main text.

C.7 Deriving estimates for ε_M

In this section, we estimate ε_M. Before we proceed more formally in estimating ε_M, recall from equation (8) in the main text that $\omega_{ijt} = \Gamma_{ij}^M p_{ijt}^{1-\varepsilon_M}$, where $p_{ijt} := P_{jt}/P_{it}^M$. As long as ω_{ijt}, Γ_{ij}^M, and p_{ijt} are all positive, we can take logarithms of both sides of this equation to obtain:

$$\log \omega_{ijt} = \log \Gamma_{ij}^M + (1-\varepsilon_M)\log p_{ijt}. \tag{C.43}$$

Clearly, there will be some industry i that does not source any inputs from industry j, so $\Gamma_{ij}^M = 0$. This logarithmic transformation is thus applicable only to those nontrivial (i,j) pairs from which we would aim to identify ε_M. Given a sample of T observations, we have

$$\frac{\sum_{t=1}^T (\log \omega_{ijt} - \overline{\log \omega_{ij}})^2}{T-1} = (1-\varepsilon_M)^2 \frac{\sum_{t=1}^T (\log p_{ijt} - \overline{\log p_{ij}})^2}{T-1}, \tag{C.44}$$

where $\overline{\log \omega_{ij}}$ and $\overline{\log p_{ij}}$ denote the time averages of $\log \omega_{ijt}$ and $\log p_{ijt}$, respectively. Denoting the two sample variances by $\hat{\sigma}^2(\log \omega_{ijt})$ and $\hat{\sigma}^2(\log p_{ijt})$, this equation can be written as

$$(1-\varepsilon_M)^2 = \frac{\hat{\sigma}^2(\log \omega_{ijt})}{\hat{\sigma}^2(\log p_{ijt})}, \tag{C.45}$$

as long as $\hat{\sigma}^2(\log p_{ijt}) > 0$.

Since the objects on the right-hand side are observable from the data, we can back out ε_M on the left-hand side. Our analysis suggests that the empirical distribution of the ratio on the right-hand side of equation (C.45) is right-skewed and has a mean (median) equal to around 0.37 (0.26).[24] Assuming that $\varepsilon_M > 0$, this implies that the

[24]We make use of the supply and use tables as well as data on industries' deflators; see Appendix A.1.

value of ε_M corresponding to the mean (median) of this distribution equals 0.40 (0.50). The estimated value of ε_M of 0.35 in our preferred specification (presented below and reproduced in Section 4.1 of the main text) is very close to 0.4, which we obtained in a cruder fashion using equation (C.45). We thus set ε_M equal to 0.35. We calibrate the remaining elasticity parameters following Atalay (2017).[25]

We now estimate ε_M more formally. As in Atalay (2017), we use industries' first-order condition with respect to intermediate inputs, and exploit their heterogeneous (direct and indirect) exposures to military spending, which we use to construct the instrument that overcomes the endogeneity of prices in the first-order condition. The cost-minimisation condition of the industry i representative firm gives the relationship between the share of intermediate inputs from industry j in industry i's total intermediate consumption:

$$\Delta \log \left(\frac{P_{jt} M_{ijt}}{P_{it}^M M_{it}} \right) = \phi_t + (1 - \varepsilon_M) \Delta \log \left(\frac{P_{jt}}{P_{it}^M} \right) + \nu_{ijt}. \tag{C.46}$$

Intuitively, assuming that military spending on industries' intermediate inputs is exogenous renders it a valid instrument, since it will be uncorrelated with the regression error term (ν_{ijt}).

Table C.1: Regression results related to equation (C.46)

Second stage	(1) OLS	(2) OLS, Year FE	(3) IV	(4) IV, Year FE
ε_M	0.680***	0.680***	0.273***	0.345**
	(0.015)	(0.015)	(0.266)	(0.263)
First stage: Dependent variable is $\Delta \log(P_{jt}/P_{it}^M)$.				
military spending shock$_{it}$			0.331***	0.385***
			(0.033)	(0.050)
military spending shock$_{jt}$			-0.205***	-0.185***
			(0.029)	(0.032)
military spending shock$_{jt\text{'s suppliers}}$			-0.051***	-0.045***
			(0.011)	(0.012)
N	35343	35343	34623	34623
Adjusted R^2	0.013	0.014	.	.
Wu-Hausman test p-value			0.124	0.204
Cragg-Donald Statistic			37.968	38.593
Year Fixed Effects	No	Yes	No	Yes

Notes: Statistical significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table C.1 shows the results of our estimation using UK data. OLS yields

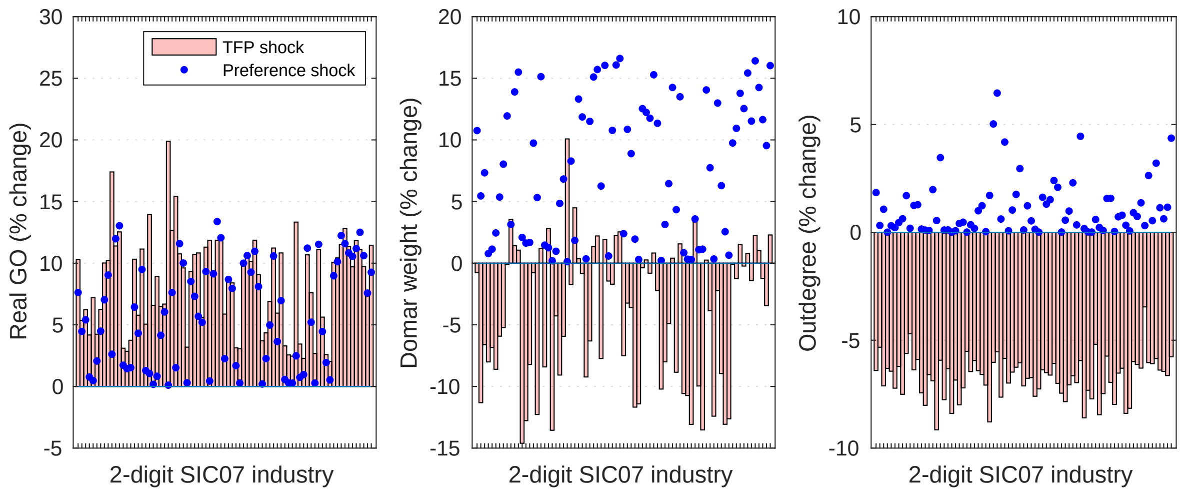
[25] Unlike in the baseline calibration in Atalay (2017), the value of ε_Q estimated or calibrated in a few other recent papers has tended to be somewhat smaller than 1. For example, Peter and Ruane (2023) estimate a value of 0.6 for the US, and Baqaee and Farhi (2019) use 0.5. Our results are generally robust to an alternative calibration with $\varepsilon_Q = 0.5$. Our results are similarly robust to reasonable alternative (non-unitary) calibrations of ε_D—see Appendix D.

estimates of ε_M around 0.68, though these estimates are inconsistent insofar as P_{jt}/P_{it}^M is endogenous in equation (C.46). IV estimation—which uses, in the first stage, three sets of instruments based on industries' heterogeneous exposures to UK military spending—yields estimates of ε_M between 0.27 and 0.35, depending on whether year fixed effects, ϕ_t, in equation (C.46) are included or not. All instruments are significant at the 1% level and the second-stage estimate of ε_M is significant at the 5% level in our preferred specification, allowing for year fixed effects.

C.8 Effects of own sector shocks

Although shocks transmit through the production network, the majority of variation in value-added and prices will result from industries' own shocks. To understand further the patterns in Figure 3 in the main text, it is therefore instructive to focus on 'own-effects' of shocks. Figure C.1 shows the contemporaneous responses of our size and centrality measures to a positive 10% technology (or preference) shock for each industry.[26,27] As in the simplified model in Section 3.2, both technology and preference shocks that are positive raise own real output. Domar weights increase in the face of own preference shocks, and generally fall in response to own technology shocks. Finally, outdegrees strongly fall in response to technology shocks, and increase (though to a lesser extent) in response to a preference shock. As industries' own shocks account for the bulk of variation in own size and centrality, these patterns thus help explain the results shown in Figure 3.

Figure C.1. Contemporaneous effect to a +10% own technology/preference shock



[26]We focus on the contemporaneous effects of shocks since the dynamics of the model feature relatively low persistence. Introducing capital adjustment costs in the model, for instance, would allow for greater persistence in the dynamic responses to shocks.

[27]Although a 10% shock is reasonable given the size of filtered innovations, scaling is not our concern since we want to illustrate the qualitative effects of these shocks; the effects of shocks are linear in the size of the shock.

D. Robustness Analysis for Section 4

In the static model, the implied size-centrality relationship—as a function of supply and demand shocks—depends crucially on the value of ε_M. Specifically, as long as ε_M was less than 1, the static model suggested supply-side shocks would move real output and centrality in opposite directions, unlike preference shocks. By varying the value of ε_M and filtering out the resulting sets of shocks (keeping other model parameters the same as in our baseline calibration), we find that the same result holds true in the estimated dynamic model (see Table D.1). With $\varepsilon_M > 1$, the effects of shocks on the size-centrality relationship generally flips sign. Although the size-centrality relationship in the data is not a targeted moment in the model filter, we nonetheless find that our baseline calibration matches it better than alternative values of ε_M shown in Table D.1. In Table D.2, we show that changing two other key parameters (ε_Q and ε_D) does not change our fundamental findings about the effects of shocks on size and centrality.

Table D.1: Implied Size Centrality Relationship in Dynamic Model Under Different Values of ε_M vs. Empirical Data Counterpart for the UK

Implied Size-Centrality Relationship	Elasticity of Substitution Across Intermediates (ε_M)							Data
	0.1	0.2	0.35 (*baseline*)	0.5	0.8	1	1.5	
$\overline{\text{corr}}(\hat{d}_t^{out}, \hat{q}_t)$, all shocks	0.03	0.02	0.00	-0.05	-0.21	-0.14	-0.27	0.12
$\overline{\text{corr}}(\hat{d}_t^{out}, \hat{\lambda}_t)$, all shocks	0.31	0.27	0.23	0.13	0.06	0.11	-0.43	0.17
$\overline{\text{corr}}(\hat{d}_t^{out}, \hat{q}_t)$, only technology shocks	-0.49	-0.54	-0.67	-0.70	-0.77	-0.76	0.03	n.a.
$\overline{\text{corr}}(\hat{d}_t^{out}, \hat{\lambda}_t)$, only technology shocks	0.73	0.69	0.58	0.46	-0.04	-0.45	0.66	n.a.
$\overline{\text{corr}}(\hat{d}_t^{out}, \hat{q}_t)$, only preference shocks	0.89	0.88	0.86	0.84	0.81	0.75	-0.59	n.a.
$\overline{\text{corr}}(\hat{d}_t^{out}, \hat{\lambda}_t)$, only preference shocks	0.92	0.91	0.90	0.90	0.88	0.84	-0.81	n.a.

Notes: $\hat{d}_t^{out}$ denotes producer centrality, $\hat{q}_t$ denotes real gross output, and $\hat{\lambda}_t$ denotes Domar weight (all in terms of steady-state log deviations).

Table D.2: Implied Size Centrality Relationship in Dynamic Model Under Different Values of ε_Q and ε_D vs. Empirical Data Counterpart for the UK

Implied Size-Centrality Relationship	Value of ε_Q			Value of ε_D			Data
	0.5	1 (*baseline*)	1.3	0.8	1 (*baseline*)	1.3	
$\overline{\text{corr}}(\hat{d}_t^{out}, \hat{q}_t)$ given all shocks	-0.18	0.00	0.19	-0.05	0.00	-0.02	0.12
$\overline{\text{corr}}(\hat{d}_t^{out}, \hat{\lambda}_t)$ given all shocks	0.06	0.23	0.41	0.21	0.23	0.25	0.17
$\overline{\text{corr}}(\hat{d}_t^{out}, \hat{q}_t)$ given only technology shocks	-0.78	-0.67	-0.52	-0.60	-0.67	-0.71	n.a.
$\overline{\text{corr}}(\hat{d}_t^{out}, \hat{\lambda}_t)$ given only technology shocks	0.61	0.58	0.49	0.76	0.58	0.19	n.a.
$\overline{\text{corr}}(\hat{d}_t^{out}, \hat{q}_t)$ given only preference shocks	0.96	0.86	0.84	0.87	0.86	0.83	n.a.
$\overline{\text{corr}}(\hat{d}_t^{out}, \hat{\lambda}_t)$ given only preference shocks	0.97	0.90	0.90	0.91	0.90	0.90	n.a.

Notes: $\hat{d}_t^{out}$ denotes producer centrality, $\hat{q}_t$ denotes real gross output, and $\hat{\lambda}_t$ denotes Domar weight (all in terms of steady-state log deviations).

E. Additional Results for Section 5

In the main text, we argued that the value of ε_M is key in determining the effect of shocks on the size-centrality relationship. Here, we find that our baseline results around the contributions of shocks to the UK's productivity growth puzzle are largely robust to varying ε_M (Table E.1). The exceptions are that we find that common and finance-specific shocks generally tend to become more important in explaining the puzzle as ε_M increases. This is to be expected, as when ε_M is higher, the model filter is more likely to attribute common variation in industries' productivity dynamics to common factors (as it is easier to substitute away from idiosyncratic sources of variation). Importantly, as the most central sector in the economy, shocks to the financial sector act much like common shocks (i.e., it is difficult to substitute away from them, even with a high ε_M). That said, the ability of the model to match the aggregate productivity growth in the data generally falls meaningfully as ε_M rises above 0.5. In Table E.2, we show that changing two other key parameters (ε_Q and ε_D) does not change our fundamental findings about the drivers of the UK's productivity growth slowdown.

Table E.1: Contributions of Sector-Specific/Common Shocks to UK's Post-2010 Productivity Growth Puzzle for Different Values of ε_M

Contribution to Productivity Growth Puzzle	Elasticity of Substitution Across Intermediates (ε_M) 0.1	0.2	0.35 (*baseline*)	0.5	0.8	1	1.3	Data
Manufacturing-specific shocks	-0.63	-0.65	-0.65	-0.60	-0.66	-0.57	-0.31	n.a.
Finance-specific shocks	-0.03	-0.03	-0.04	-0.07	-0.08	-0.11	-0.16	n.a.
Other sectors' specific shocks	0.30	0.30	0.30	0.41	0.35	0.55	0.23	n.a.
Common shocks	0.12	0.13	0.13	-0.01	0.10	-0.13	-0.22	n.a.
Total growth puzzle	-0.24	-0.24	-0.26	-0.27	-0.28	-0.27	-0.46	-0.18
Correlation of model-implied aggregate productivity growth and data counterpart	0.79	0.78	0.77	0.75	0.70	0.62	0.25	n.a.

Notes: All other model parameters set according to the baseline calibration, shown in Table 2 in the main text.

Table E.2: Contributions of Sector-Specific/Common Shocks to UK's Post-2010 Productivity Growth Puzzle for Different Values of ε_Q and ε_D

Contribution to Productivity Growth Puzzle	Value of ε_Q 0.5	1 (*baseline*)	1.3	Value of ε_D 0.8	1 (*baseline*)	1.3	Data
Manufacturing-specific shocks	-0.54	-0.65	-0.66	-0.65	-0.65	-0.66	n.a.
Finance-specific shocks	-0.00	-0.04	-0.09	-0.02	-0.04	-0.06	n.a.
Other sectors' specific shocks	0.15	0.30	0.40	0.35	0.30	0.25	n.a.
Common shocks	0.15	0.13	0.09	0.16	0.13	0.12	n.a.
Total growth puzzle	-0.24	-0.26	-0.26	-0.15	-0.26	-0.35	-0.18
Correlation of model-implied aggregate productivity growth and data counterpart	0.76	0.77	0.74	0.75	0.77	0.76	n.a.

Notes: All other model parameters set according to the baseline calibration, shown in Table 2 in the main text.

Figure E.1. Contributions to the Growth Puzzle: Sectors vs. Sectoral Shocks (Dashed)

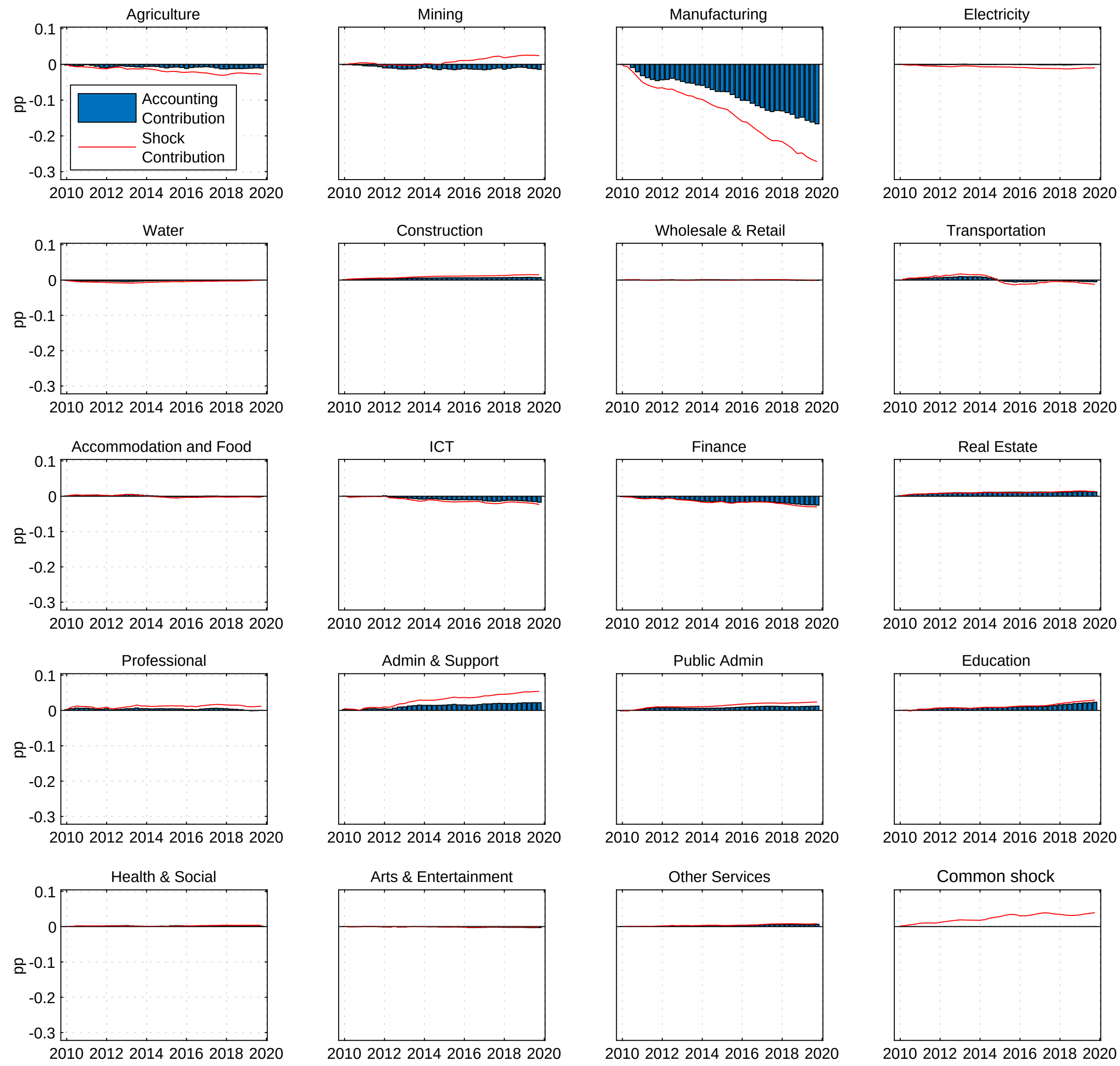


Figure E.2. Robustness of Baseline Results in Figure 7 in Alternative Parametrisations